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Nanoscale control over optical singularities: supplementary material

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This document provides supplementary information to "Nanoscale control over optical singularities," https://doi.org/10.1364/OPTICA.5.000283. Provided below is additional information regarding the main parts of the analytical derivation of our method: control over the polarization state, plasmonic in-plane field derivation, polarization state, and analysis of pure high order OV state.

1. CONTROL OVER THE POLARIZATION STATE

Amplitude ratio and phase difference between two orthogonal circular polarizations can be derived using a simple calculation of plane wave propagating through Quarter Wave (QW) and Half Wave (HW) plates.

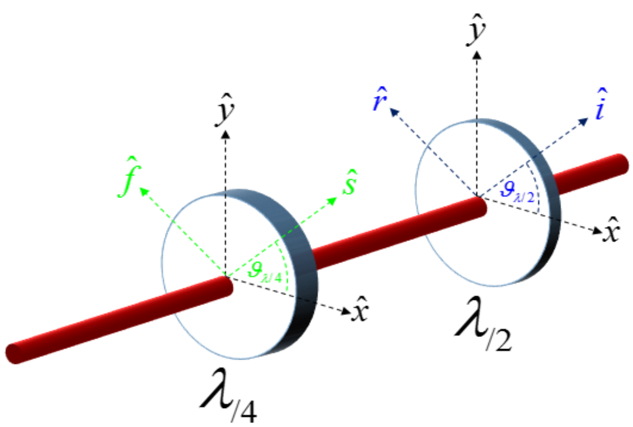

Fig. 1S. Coordinate systems. Schematic representation of the free space coordinate system $\hat{x}, \hat{y}$. The fast and the slow axis of the QW and HW plates represented with $\hat{f}, \hat{s}$ and $\hat{r}, \hat{\imath}$ respectively.

We start with the linearly-polarized field in the $\hat{x}$ direction launched on the QW plate:

$$\vec{E}_{in,\lambda/4} = \hat{x} = \sin\vartheta_{\lambda/4}\hat{s} + \cos\vartheta_{\lambda/4}\hat{f} \qquad \textbf{(S1)}$$

Note that the coordinate system transformation used is:

$$\begin{cases}\hat{x} = \cos\vartheta_{\lambda/4}\hat{f} + \sin\vartheta_{\lambda/4}\hat{s}\\ \hat{y} = \sin\vartheta_{\lambda/4}\hat{f} - \cos\vartheta_{\lambda/4}\hat{s}\end{cases} \qquad \textbf{(S2)}$$

After propagating through the QW plate, the field is of the form:

(S3)

$$\vec{E}_{out,\lambda/4} = \sin\vartheta_{\lambda/4}\hat{s} + i\cos\vartheta_{\lambda/4}\hat{f} = \left(\sin^2\vartheta_{\lambda/4} + i\cos^2\vartheta_{\lambda/4}\right)\hat{x} - \cos\vartheta_{\lambda/4}\sin\vartheta_{\lambda/4}(1-i)\hat{y} = a\hat{x} + b\hat{y} (3)$$

We then proceed with the field entering the HW plate:

(S4)

$$\{\vec{E}_{in,\lambda/2} = a\hat{x} + b\hat{y} = \left(a\sin\vartheta_{\lambda/2} - b\cos\vartheta_{\lambda/2}\right)\hat{r} + \left(a\cos\vartheta_{\lambda/2} + b\sin\vartheta_{\lambda/2}\right)\hat{\imath} = c\hat{r} + d\hat{\imath}$$

Note that the coordinate system transformation used is:

$$\begin{cases}\hat{x} = \cos\vartheta_{\lambda/2}\hat{\imath} + \sin\vartheta_{\lambda/2}\hat{r}\\ \hat{y} = \sin\vartheta_{\lambda/2}\hat{\imath} - \cos\vartheta_{\lambda/2}\hat{r}\end{cases} \qquad \textbf{(S5)}$$

The field at the output of the HW plate is now:

(S6)

$$\vec{E}_{out,\lambda/2} = c\hat{r} - d\hat{\imath} = \hat{x}\left(c\sin\vartheta_{\lambda/2} - d\cos\vartheta_{\lambda/2}\right) + \hat{y}\left(-c\cos\vartheta_{\lambda/2} - d\sin\vartheta_{\lambda/2}\right) = \hat{x}e + \hat{y}f$$

Now we write the last field as a superposition of left and right circular polarizations:

$$\vec{E}_{out,\lambda/2} = \hat{x}e + \hat{y}f = g(\hat{x} + i\hat{y}) + h(\hat{x} - i\hat{y}) \qquad \textbf{(S7)}$$

Where $g = \frac{f+ie}{2i}$, $h = \frac{f-ie}{-2i}$.

Now the relative amplitude and phase between the circular polarizations can be calculated simply as:

$$f_{a_0}(\vartheta_{\lambda/4}, \vartheta_{\lambda/2}) = \left|\frac{g}{h}\right|; f_{\varphi_0}(\vartheta_{\lambda/4}, \vartheta_{\lambda/2}) = \arg\left(\frac{g}{h}\right) \quad \textbf{(S8)}$$

2. IN PLANE FIELD

As we previously stated in the manuscript, the vectorial Electric field field consists of the out-of-plane field component E_z and the in-plane field components E_ρ, E_θ as depicted in Fig.2S.

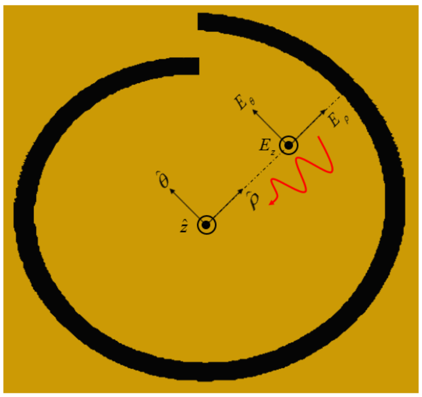

Fig. 2S. SPPs platform. Schematic representation of the plasmon and its electrical field.

The In-plane field components: E_ρ, E_θ can be derived from the out-of-plane field E_z using Maxwell's equations in cylindrical coordinates and prior knowledge about the TM nature of the SPPs:

(S9)

$$\nabla \times E = \left(\frac{1}{\rho}\frac{\partial}{\partial \theta}E_z - \frac{\partial}{\partial z}E_\theta\right)\hat{\rho} + \left(\frac{\partial}{\partial z}E_\rho - \frac{\partial}{\partial \rho}E_z\right)\hat{\varphi} + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}\rho E_\theta - \frac{\partial}{\partial \theta}E_\rho\right)\hat{z}$$

$$\downarrow \nabla \times E = -i\omega\mu H$$

$$\begin{cases} \frac{1}{\rho}\frac{\partial}{\partial \theta}E_z - \frac{\partial}{\partial z}E_\theta = -i\omega\mu H_\rho \\ \frac{\partial}{\partial z}E_\rho - \frac{\partial}{\partial \rho}E_z = -i\omega\mu H_\theta \\ \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho E_\theta - \frac{1}{\rho}\frac{\partial}{\partial \theta}E_\rho = -i\omega\mu H_z \end{cases}$$

(S10)

$$\nabla \times H = \left(\frac{1}{\rho}\frac{\partial}{\partial \theta}H_z - \frac{\partial}{\partial z}H_\theta\right)\hat{\rho} + \left(\frac{\partial}{\partial z}H_\rho - \frac{\partial}{\partial \rho}H_z\right)\hat{\theta} + \frac{1}{\rho}\left(\frac{\partial}{\partial \rho}\rho H_\theta - \frac{\partial}{\partial \theta}H_\rho\right)\hat{z}$$

$$\downarrow \nabla \times H = i\omega\varepsilon E$$

$$\begin{cases} \frac{1}{\rho}\frac{\partial}{\partial \theta}H_z - \frac{\partial}{\partial z}H_\theta = i\omega\varepsilon E_\rho \\ \frac{\partial}{\partial z}H_\rho - \frac{\partial}{\partial \rho}H_z = i\omega\varepsilon E_\theta \\ \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho H_\theta - \frac{1}{\rho}\frac{\partial}{\partial \theta}H_\rho = i\omega\varepsilon E_z \end{cases}$$

Using the fact that SPPs only have TM polarization, we conclude that: $H_z = 0$. As stated in the manuscript, the field decays exponentially in the out-of-plane direction z, namely, $\frac{\partial}{\partial z} \to \pm i\beta_z$ where $\beta_z = k_z$ is the wave number of the SPPs in the decaying direction. We then derive that the in-plane field:

$$E_\rho = \xi\frac{\partial}{\partial \rho}E_z;\ E_\theta = \xi\frac{1}{\rho}\frac{\partial}{\partial \theta}E_z \quad \textbf{(S11)}$$

Where $\xi = \frac{\sqrt{\varepsilon_d/\varepsilon_m}}{k_{sp}}$, $k_{sp} = k_0\frac{\varepsilon_m \cdot \varepsilon_d}{\varepsilon_m + \varepsilon_d}$ is the wave vector of the SPPs in the propagation direction and $\varepsilon_m, \varepsilon_d$ are the permittivity of the metal and the air respectively.

3. POLARIZATION STATE

The polarization ellipse coefficients: ellipse orientation α and eccentricity ε can be calculated [1] via the in-plane-field components as:

(S12)

$$\alpha = \frac{1}{2}\tan\left(\frac{2|E_x||E_y|cos(\delta)}{|E_x|^2 - |E_y|^2}\right);\ \varepsilon = \frac{(|E_x|sin(\alpha))^2 + (|E_y|cos(\alpha))^2 - |E_x||E_x|cos(\alpha)sin(2\alpha)}{(|E_x|cos(\alpha))^2 + (|E_y|sin(\alpha))^2 + |E_x||E_x|cos(\alpha)sin(2\alpha)}$$

Where $\delta = phase(E_x) - phase(E_y)$ is the phase difference between the field components and E_x, E_y are the Cartesian representation of the in plane field components E_ρ, E_θ which can be derived simply by the coordinate system transformation:

$$\begin{cases} E_{x(\rho,\theta)} = E_{\rho(\rho,\theta)}\cos\theta - E_{\theta(\rho,\theta)}\sin\theta \\ E_{y(\rho,\theta)} = E_{\rho(\rho,\theta)}\sin\theta + E_{\theta(\rho,\theta)}\cos\theta \end{cases} \quad \textbf{(S13)}$$

Where (ρ, θ) are the polar coordinates of the field in a specific point.

4. PUREST HIGH ORDER OV

Our experimental platform is limited in generating a pure high order OV, which always breaks up into a series of vortices. The coupling slit is asymmetrical and thus the optical path length from each point in the slit is different, resulting in an amplitude difference between the various interfering plasmonic waves. This, in turn, induces additional high order Bessel modes, causing a vortex break up. We analyze the purest possible state of OVs of order 2, 3 by the minimal distance between the newly-created unity OVs that can be achieved with our chosen slit.

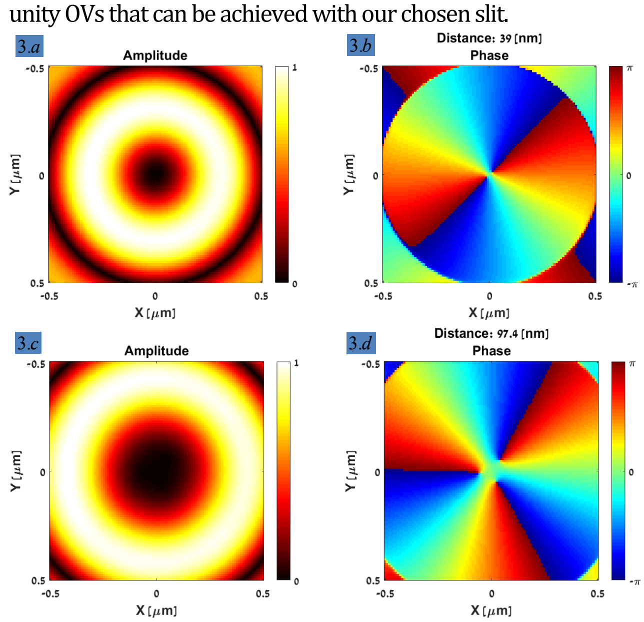

Fig. 3S. Purest state simulation. Simulation of purest state of high order OVs of order 2 (upper row) and 3 (bottom row). Amplitude (a, c) and phase (b, d) are presented. The minimal distance between the unity OVs is indicated above the phase images.

As can be seen, the minimal distance between OVs is around 40nm for $l = 1$ and 100nm for $l = 2$.

REFERENCES

1. A. D. Hoogh et al. “Optical singularities and nonlinear effects near plasmonic nanostructures,” Ph.D. thesis (Delft University of Technology, 2016), pp. 13-14.