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Stable single light bullets and vortices and their active control in cold Rydberg gases: supplementary material

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1. THE DYNAMICS OF THE RYDBERG ATOM

In this section, we provide the theory of the many-body dynamics in cold Rydberg gases. We will focus on the derivation of the hierarchy of equations for many-body correlators provided in the main text and its analytical solution.

A. Optical Bloch equation

We first treat the dynamics under the effective Hamiltonian in the Heisenberg picture. To this end we obtain the equation of motion for the expectation values $\hat{\rho}$ from the corresponding operator Heisenberg equations. This yields the explicit expression of the Bloch equation

$$i\frac{\partial}{\partial t}\rho_{11} - i\Gamma_{12}\rho_{22} - \Omega_p\rho_{12} + \Omega_p^*\rho_{21} = 0, \tag{S1a}$$

$$i\frac{\partial}{\partial t}\rho_{22} - i\Gamma_{23}\rho_{33} + i\Gamma_{12}\rho_{22} + \Omega_p\rho_{12} - \Omega_p^*\rho_{21} - \Omega_c\rho_{23} + \Omega_c^*\rho_{32} = 0, \tag{S1b}$$

$$i\frac{\partial}{\partial t}\rho_{33} + i\Gamma_{23}\rho_{33} + \Omega_c\rho_{23} - \Omega_c^*\rho_{32} = 0, \tag{S1c}$$

for diagonal elements, and

$$\left(i\frac{\partial}{\partial t} + d_{21}\right)\rho_{21} - \Omega_p(\rho_{22} - \rho_{11}) + \Omega_c^*\rho_{31} = 0, \tag{S2a}$$

$$\left(i\frac{\partial}{\partial t} + d_{31}\right)\rho_{31} - \Omega_p\rho_{32} + \Omega_c\rho_{21} - \mathcal{N}_a\int d^3\mathbf{r}' V(\mathbf{r}'-\mathbf{r})\rho_{33,31}(\mathbf{r}',\mathbf{r},t) = 0, \tag{S2b}$$

$$\left(i\frac{\partial}{\partial t} + d_{32}\right)\rho_{32} - \Omega_p^*\rho_{31} - \Omega_c(\rho_{33} - \rho_{22}) - \mathcal{N}_a\int d^3\mathbf{r}' V(\mathbf{r}'-\mathbf{r})\rho_{33,32}(\mathbf{r}',\mathbf{r},t) = 0, \tag{S2c}$$

for non-diagonal elements. Here $\rho_{\alpha\beta}(\mathbf{r},t) \equiv \langle \hat{S}_{\alpha\beta}(\mathbf{r},t)\rangle$ are the one-body correlators (one-body density matrix elements), $d_{\alpha\beta} = \Delta_\alpha - \Delta_\beta + i\gamma_{\alpha\beta}$ ($\Delta_1 = 0$; $\alpha,\beta = 1,2,3; \alpha \neq \beta$), $\Delta_2 = \omega_p - (\omega_2 - \omega_1)$ and $\Delta_3 = \omega_p + \omega_c - (\omega_3 - \omega_1)$ are respectively the one-photon and two-photon detunings, $\gamma_{\alpha\beta} = (\Gamma_\alpha + \Gamma_\beta)/2 + \gamma_{\alpha\beta}^{\rm col}$ with $\Gamma_\beta = \sum_{\alpha<\beta}\Gamma_{\alpha\beta}$. Here $\Gamma_{\alpha\beta}$ denotes respectively the spontaneous emission decay rate from the state $|\beta\rangle$ to the state $|\alpha\rangle$, and $\gamma_{\alpha\beta}^{\rm col}$ represents the dephasing rate reflecting the loss of phase coherence between $|\alpha\rangle$ and $|\beta\rangle$ due to, e.g., atomic motion and the interaction between the atoms in the ground and Rydberg states.

The last terms on the left hand side of Eq. (S2b) and Eq. (S2c), i.e. the two-body correlators $\rho_{33,3\alpha}(\mathbf{r}',\mathbf{r},t) \equiv \langle \hat{S}_{33}(\mathbf{r}',t)\hat{S}_{3\alpha}(\mathbf{r},t)\rangle$ ($\alpha = 1,2$), are contributed from the interaction between Rydberg atoms, where attractive (repulsive) atomic interaction lead to nonlocal self-focusing (self-defocusing) Kerr nonlinearities.

Besides, there is another type of (local) nonlinearity when two-photon detuning is non-zero (i.e. $\Delta_3 \neq 0$) [1], contributed solely from the resonant coupling between the probe field and atoms.

B. Equations and solutions for the two-body correlators

When deriving the nonlinear envelope equation (3) in the main text, one needs to solve the equations of motion for the two-body correlators $\rho_{\alpha\beta,\mu\nu}$, which have no contribution at the first order approximation [1, 2]. At second- and third-order, these equations are given as follows:

(i) *Second-order approximation*. We have the equation

$$\begin{bmatrix} 2\omega+2d_{21} & 0 & 2\Omega_c^* \\ 0 & 2\omega+2d_{31}-V & 2\Omega_c \\ \Omega_c & \Omega_c^* & 2\omega+d_{21}+d_{31} \end{bmatrix} \begin{bmatrix} \rho_{21,21}^{(2)} \\ \rho_{31,31}^{(2)} \\ \rho_{31,21}^{(2)} \end{bmatrix} = \begin{bmatrix} -2\frac{\omega+d_{31}}{D(\omega)} \\ 0 \\ \frac{\Omega_c}{D(\omega)} \end{bmatrix} F^2 e^{2i\theta}, \tag{S3}$$

$$\begin{bmatrix} d_{21}+d_{12} & 0 & -\Omega_c & \Omega_c^* \\ -\Omega_c^* & \Omega_c^* & d_{21}+d_{13} & 0 \\ 0 & d_{31}+d_{13} & \Omega_c & -\Omega_c^* \\ -\Omega_c & \Omega_c & 0 & d_{21}^*+d_{13}^* \end{bmatrix} \begin{bmatrix} \rho_{21,12}^{(2)} \\ \rho_{31,13}^{(2)} \\ \rho_{21,13}^{(2)} \\ \rho_{21,13}^{*(2)} \end{bmatrix} = \begin{bmatrix} \frac{\omega+d_{31}}{D} - \frac{\omega+d_{31}^*}{D(\omega)^*} \\ \frac{\Omega_c^*}{D(\omega)^*} \\ 0 \\ \frac{\Omega_c}{D(\omega)} \end{bmatrix} |F|^2 e^{-2\bar{\alpha} z_2}. \tag{S4}$$

The solution of the equation is given by $\rho_{\alpha 1,\beta 1}^{(2)} = a_{\alpha 1,\beta 1}^{(2)} F^2 e^{2i\theta}$, $\rho_{\alpha 1,1\beta}^{(2)} = a_{\alpha 1,1\beta}^{(2)} |F|^2 e^{-2\bar{\alpha} z_2}$ $(\alpha,\beta = 2,3)$. Note that for weak nonlinear probe pulse the atom-atom interaction represented by the vdW potential $\hbar V$ has no contribution to $\rho_{\alpha 1,1\beta}^{(2)}$, thus we have $\rho_{\alpha 1,1\beta}^{(2)} = \rho_{\alpha 1}^{(1)} \rho_{1\beta}^{(1)}$.

(ii) *Third-order approximation*. We have the equation

$$\begin{bmatrix} M_{31} & \Omega_c^* & -i\Gamma_{23} & 0 & \Omega_c^* & -\Omega_c & 0 & 0 \\ \Omega_c & M_{32} & 0 & -i\Gamma_{23} & 0 & 0 & \Omega_c^* & -\Omega_c \\ 0 & 0 & M_{33} & \Omega_c^* & -\Omega_c^* & \Omega_c & 0 & 0 \\ 0 & 0 & \Omega_c & M_{34} & 0 & 0 & -\Omega_c^* & \Omega_c \\ \Omega_c & 0 & -\Omega_c & 0 & M_{35} & 0 & \Omega_c^* & 0 \\ -\Omega_c^* & 0 & \Omega_c^* & 0 & 0 & M_{36} & 0 & \Omega_c^* \\ 0 & \Omega_c & 0 & -\Omega_c & \Omega_c & 0 & M_{37} & 0 \\ 0 & -\Omega_c^* & 0 & \Omega_c^* & 0 & \Omega_c & 0 & M_{38} \end{bmatrix} \begin{bmatrix} \rho_{22,21}^{(3)} \\ \rho_{22,31}^{(3)} \\ \rho_{33,21}^{(3)} \\ \rho_{33,31}^{(3)} \\ \rho_{32,21}^{(3)} \\ \rho_{21,23}^{(3)} \\ \rho_{32,31}^{(3)} \\ \rho_{31,23}^{(3)} \end{bmatrix} = \begin{bmatrix} -a_{21,12}^{(2)} + a_{21,21}^{(2)} - a_{22}^{(2)} \\ -a_{31,12}^{(2)} + a_{21,31}^{(2)} \\ -a_{33}^{(2)} \\ 0 \\ a_{21,31}^{(2)} - a_{32}^{(2)} \\ -a_{32}^{*(2)} - a_{21,13}^{(2)} \\ a_{31,31}^{(2)} \\ -a_{31,13}^{(2)} \end{bmatrix} |F(\mathbf{r}')|^2 F(\mathbf{r}) e^{-2\bar{\alpha} z_2'} e^{i\theta}, \tag{S5}$$

where $M_{31} = \omega + i\Gamma_{12} + d_{21}$, $M_{32} = \omega + i\Gamma_{12} + d_{31}$, $M_{33} = \omega + i\Gamma_{23} + d_{21}$, $M_{34} = \omega + d_{31} + i\Gamma_{23} - V$, $M_{35} = \omega + d_{32} + d_{21}$, $M_{36} = \omega + d_{23} + d_{21}$, $M_{37} = \omega + d_{32} + d_{31} - V$ and $M_{38} = \omega + d_{23} + d_{31}$. Solving this equation we obtain the third order solution

$$\rho_{33,31}^{(3)} = a_{33,31}^{(3)} |F(\mathbf{r}')|^2 F(\mathbf{r}) e^{-2\bar{\alpha} z_2'} e^{i\theta}, \tag{S6}$$

with

$$a_{33,31}^{(3)} = \frac{P_0 + P_1 V(\mathbf{r}'-\mathbf{r}) + P_2 V(\mathbf{r}'-\mathbf{r})^2}{Q_0 + Q_1 V(\mathbf{r}'-\mathbf{r}) + Q_2 V(\mathbf{r}'-\mathbf{r})^2 + Q_3 V(\mathbf{r}'-\mathbf{r})^3}, \simeq \frac{-2|\Omega_c|^2 \Omega_c (2\omega + d_{21} + d_{31})/|D(\omega)|^2}{2(\omega+d_{21})|\Omega_c|^2 - D_2(\omega)[2\omega + 2d_{31} - V(\mathbf{r}'-\mathbf{r})]}. \tag{S7}$$

Here $D_2(\omega) = (\omega + d_{21})(2\omega + d_{21} + d_{31}) - |\Omega_c|^2$, P_n and Q_n $(n = 0,1,2,3)$ are functions of the spontaneous emission decay rate $\gamma_{\mu\nu}$, detunings Δ_μ, and half Rabi frequency Ω_c (they are lengthy thus not written explicitly down here).

2. DERIVATION OF THE NONLINEAR ENVELOPE EQUATION.

Here we give a simple description on the derivation of the (3+1)D nonlinear envelope equation (3) in the main text by employing the method of multiple-scales [3, 4]. To this end, we assume that the atoms are initially populated in state $|1\rangle$ and so can make the asymptotic expansion: $\rho_{\alpha 1} = \sum_{l=0} \epsilon^{2l+1} \rho_{\alpha 1}^{(2l+1)}$, $\rho_{32} = \sum_{l=1} \epsilon^{2l} \rho_{32}^{(2l)}$, $\rho_{\beta\beta} = \sum_{l=0} \epsilon^{2l} \rho_{\beta\beta}^{(2l)}$ with $\rho_{\beta\beta}^{(0)} = \delta_{\beta 1}\delta_{\beta 1}(\alpha = 2,3; \beta = 1,2,3)$, $\Omega_p = \sum_{l=1} \epsilon^l \Omega_p^{(l)}$, where ϵ a parameter characterizing the magnitude of Ω_p. To obtain a divergence-free expansion, all quantities on the right hand side of the expansion are considered as functions of the multiscale variables $z_l = \epsilon^l z$ $(l = 0,1,2)$, $(x_1, y_1) = \epsilon(x,y)$, and $t_l = \epsilon^l t$ $(l = 0,1)$. Substituting the above expansion into the Maxwell-Bloch Equations, and comparing the coefficients of ϵ^l $(l = 1,2,...)$, we obtain a set of linear but inhomogeneous equations which can be solved order by order.

At the first order, we obtain the solution $\Omega_p^{(1)} = F\exp[i(Kz_0 - \omega t_0)]$, $\rho_{21}^{(1)} = (\omega + d_{31})\Omega_p^{(1)}/D$, and $\rho_{31}^{(1)} = -\Omega_c\Omega_p^{(1)}/D$, with $D = |\Omega_c|^2 - (\omega + d_{21})(\omega + d_{31})$ and other $\rho_{\alpha\beta}^{(1)}$ to be zero. In above expressions, $K(\omega)$ is the linear dispersion relation given by $K(\omega) = \omega/c + \kappa_{12}(\omega + d_{31})/D$, and F is an envelope function of the slow variables x_1, y_1, z_1, t_1, and z_2, to be determined yet.

At the second order, we obtain the equation

$$i\left(\frac{\partial F}{\partial z_1} + \frac{1}{V_g}\frac{\partial F}{\partial t_1}\right) = 0, \tag{S8}$$

with $V_g = (\partial K/\partial\omega)^{-1}$ the group velocity of the envelope F. Explicit expressions of the second-order solution for $\rho_{\alpha\beta}^{(2)}$ read

$$a_{21}^{(2)} = \frac{i}{\kappa_{12}}\left(\frac{1}{V_g} - \frac{1}{c}\right), \tag{S9a}$$

$$a_{31}^{(2)} = -\frac{i}{\Omega_c^*}\frac{\omega + d_{31}}{D} - \frac{(\omega + d_{21})}{\Omega_c^*}a_{21}^{(2)}, \tag{S9b}$$

$$a_{11}^{(2)} = \frac{[i\Gamma_{23} - 2|\Omega_c|^2 M_1]M_2 - i\Gamma_{12}|\Omega_c|^2 M_3}{-\Gamma_{12}\Gamma_{23} - i\Gamma_{12}|\Omega_c|^2 M_1}, \tag{S9c}$$

$$a_{33}^{(2)} = \frac{1}{i\Gamma_{12}}\left(M_2 - i\Gamma_{12}a_{11}^{(2)}\right), \tag{S9d}$$

$$a_{32}^{(2)} = \frac{1}{d_{32}}\left(-\frac{\Omega_c}{D} + 2\Omega_c a_{33}^{(2)} + \Omega_c a_{11}^{(2)}\right), \tag{S9e}$$

where $\rho_{\alpha 1}^{(2)} = a_{\alpha 1}^{(2)}\partial F/\partial t_1 \exp(i\theta)$, $\rho_{32}^{(2)} = a_{32}^{(2)}|F|^2\exp(-2\bar{\alpha}z_2)$, $\rho_{\beta\beta}^{(2)} = a_{\beta\beta}^{(2)}|F|^2\exp(-2\bar{\alpha}z_2)(\alpha = 2,3; \beta = 1,2,3)$ with $\theta = Kz_0 - \omega t_0$, $\bar{\alpha} = \epsilon^{-2}\alpha = \epsilon^{-2}\mathrm{Im}(K)$, $M_1 = 1/d_{32} - 1/d_{32}^*$, $M_2 = (\omega + d_{31}^*)/D^* - (\omega + d_{31})/D$, and $M_3 = 1/(D^* d_{32}^*) - 1/(D d_{32})$. Expressions of equations and solutions of the two-body correlators $\rho_{\alpha\beta,\mu\nu}^{(2)}$ are presented in Sec. 1B of the Supplementary Material.

With the above solutions, we can go to the third order. We obtain the following equation

$$\begin{aligned} &i\frac{\partial F}{\partial z_2} - \frac{1}{2}K_2\frac{\partial^2 F}{\partial t_1^2} + \frac{c}{2\omega_p}\left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial y_1^2}\right)F - W_1|F|^2 F e^{-2\bar{\alpha}z_2} \\ &+ \frac{\kappa_{12}\mathcal{N}_\alpha\Omega_c^*}{D}\left(\int d^3\mathbf{r}' a a_{33,31}^{(3)} V(\mathbf{r}' - \mathbf{r})|F(\mathbf{r}')|^2\right)F(\mathbf{r})e^{-2\bar{\alpha}z_2} \\ &= 0. \end{aligned} \tag{S10}$$

Combined Eqs. (S8) and (S10) and returning to the original variables, we obtain the Eq. (3) in the main text. Note that when obtaining Eq. (3), for simplicity we have assumed that the spatial length of the probe pulse in the propagation (i.e., z) direction is much larger than the range of Rydberg-Rydberg interactions, so that a local approximation along the z direction can be made [2].

3. OPTICAL SOLITONS IN THE LOCAL RESPONSE REGION

Note that under the EIT condition (i.e. $|\Omega_c|^2 > \gamma_{21}\gamma_{31}$) and a large one-photon detuning Δ_2, imaginary parts of the coefficients in Eq. (3) in the main text are very small. On the other hand, as indicated in the main text, in the local response region the nonlocal Kerr nonlinearity is reduced into a local one. As a result, Eq. (3) can be written as the following form

$$i\frac{\partial u}{\partial s} - s_d\frac{\partial^2 u}{\partial\sigma^2} + 2u|u|^2 = -g_{\mathrm{diff}}\left(\frac{\partial^2}{\partial\xi^2} + \frac{\partial^2}{\partial\eta^2}\right)u + id_0 u, \tag{S11}$$

with $s = z/(2L_D)$, $\sigma = \tau/\tau_0$, $(\xi,\eta) = (x,y)/R_0$, $u = U/U_0$, $g_{\mathrm{diff}} = L_D/L_{\mathrm{diff}}$, $d_0 = -2L_D/L_A$, and $s_d = \mathrm{sgn}(K_2) = \pm 1$. Here $L_{\mathrm{diff}} \equiv (\omega_p R_0^2)/c$, $L_D \equiv \tau_0{}^2/|\tilde{K}_2|$ and $L_A \equiv 1/\alpha_0$ the typical diffraction length, dispersion length, and absorption length, respectively. Note that we have taken $L_D = L_{\mathrm{NL}}$ [with $L_{\mathrm{NL}} \equiv 1/(U_0^2\tilde{W}_1 + U_0^2\tilde{W}_2)$ being a typical nonlinear length], i.e., a balance of dispersion and nonlinearity is assumed to favor the formation of solitons, thus the typical Rabi frequency of the probe field is given by $U_0 \equiv (1/\tau_0)[|\tilde{K}_2/(\tilde{W}_1 + \tilde{W}_2)|]^{1/2}$. The tilde above corresponding quantities means taking their real parts. Due to the EIT effect and large Δ_2, we have $d_0 \ll 1$, hence the dissipation plays a negligible role; furthermore, due to large R_0 we have L_{diff} ($\gg L_{\mathrm{D}}$), and thus $g_{\mathrm{diff}} \ll 1$, which means that the diffraction in the system can also be neglected. Ignoring the terms on RHS of Eq. (S11) and converting to the original variables, we obtain the bright soliton solution

$$\Omega_p = \frac{1}{\tau_0}\sqrt{\frac{|\tilde{K}_2|}{\tilde{W}}}\mathrm{sech}\left[\frac{1}{\tau_0}\left(t - \frac{z}{\tilde{V}_g}\right)\right]e^{i\tilde{K}_0 z - iz/2L_{\mathrm{D}}}, \tag{S12}$$

if $K_2 < 0$ (i.e. $s_d = -1$), with $\tilde{K}_0 = \tilde{K}|_{\omega=0}$.

Choosing system parameters the same as those used in Fig. 2(a), we obtain the numerical values of the coefficients in Eq. (S11), given by $K_2 = (-1.03 + 0.06i) \times 10^{-13}\mathrm{cm}^{-1}\mathrm{s}^2$, $W_1 = (6.86 + 0.062i) \times 10^{-18}\mathrm{cm}^{-1}\mathrm{s}^2$, $W_2 = (1.79 + 0.0076i) \times 10^{-14}\mathrm{cm}^{-1}\mathrm{s}^2$. We see that the imaginary parts of these coefficients are indeed much smaller than their real parts. Taking $\tau_0 = 1.2 \times 10^{-7}$ s, we obtain $U_0 = 2 \times 10^7\,\mathrm{s}^{-1}$, $L_D = L_{\mathrm{NL}} = 1.4$ mm, $L_A = 560$ mm, and $d_0 = -0.005$, which means that the dissipation effect of the system is indeed small. Additionally, because the beam radius of the probe pulse is large ($R_0 = 300$¯m), one has $L_{\mathrm{diff}} = 1226$ mm and hence $g_{\mathrm{diff}} = 0.001$, so the diffraction effect [the first term on the RHS of Eq. (S11)] in the system can indeed be neglected.

With the system parameters given above, it is easy to obtain $\tilde{V}_g = 9.8 \times 10^{-5}c$, i.e. the soliton obtained has an ultraslow propagation velocity compared with c, the light speed in vacuum. Furthermore, the third-order nonlinear optical susceptibility, given by $\chi_p^{(3)} = 2c|\mathbf{p}_{12}|^2(W_1 + W_2)/(\hbar^2\omega_p)$, is estimated to have the value $(3.03 + 0.0129i) \times 10^{-8}\,\mathrm{m}^2\,\mathrm{V}^{-2}$, which is 11 orders of magnitude higher than that obtained in conventional nonlinear optical media [5]. The physical reason for such large third-order nonlinear optical susceptibility is due to the strong Rydberg-Rydberg interaction and the EIT effect in the system. By using Poynting's vector [3, 5], the maximum average power density to generate such ultraslow optical soliton is estimated to be $\bar{P}_{\max} = 1.2\,\mu$W. Thus, very low input power is needed for generating such optical soliton in the system.

4. DEFINITIONS OF THE EFFICIENCY AND FIDELITY FOR OPTICAL PULSE MEMORY

The memory quality of the LBs and LVs can be described by the efficiency and fidelity of the memory. Following Ref. [6], the efficiency of a memory is described by the energy ratio between the retrieved pulse and the input pulse, i.e., $\eta = I_{\mathrm{out}}/I_{\mathrm{in}}$, where $I_{\mathrm{out}} = \int_{T_{\mathrm{on}}}^{\infty} dt \iint dxdy\,|\Omega_p^{\mathrm{out}}(x,y,t)|^2$, $I_{\mathrm{in}} =$

$\int_{-\infty}^{T_{\rm off}} dt \iint dxdy\, |\Omega_p^{\rm in}(x,y,t)|^2$, $\Omega_p^{\rm in}(x,y,t) = \Omega_p(x,y,z,t)|_{z=0}$ and $\Omega_p^{\rm out}(x,y,t) = \Omega_p(x,y,z,t)|_{z=L_z}$, with L_z the medium length in the propagation direction.

The fidelity of the memory characterizes the preservation of the waveshape of the probe pulse when it is stored first and then retrieved, which is defined by $\zeta = \eta J^2$, with $J^2 = I_{\rm inter}^2/(I_{\rm in} I_{\rm out})$. Here $I_{\rm inter} = |\int_{-\infty}^{T_{\rm off}} dt \iint dxdy\, \Omega_p^{\rm out}(x,y,t+\Delta T)\Omega_p^{\rm in}(x,y,t)|$, ΔT is the time interval between the peaks of the input and the retrieved probe pulses [6].

REFERENCES

1. Z. Bai, and G. Huang, “Enhanced third-order and fifth-order Kerr nonlinearities in a cold atomic system via Rydberg-Rydberg interaction,” Opt. Express **24**, 4442 (2016).
2. S. Sevinçli,N. Henkel,C. Ates, and T. Pohl, “Nonlocal Nonlinear Optics in Cold Rydberg Gases,” Phys. Rev. Lett. **107**, 153001 (2011).
3. G. Huang, L. Deng, and M. G. Payne, “Dynamics of ultra-slow optical solitons in a cold three-state atomic system,” Phys. Rev. E **72**, 016617 (2005).
4. A. Jeffery and T. Kawahawa, “Asymptotic Method in Nonlinear Wave Theory” (Pitman, London, 1982).
5. A. C. Newell, and J. V. Moloney, “Nonlinear Optics” (Addison-Wesley, Redwood City, 1992).
6. I. Novikova, N. B. Phillips, and A. V. Gorshkov, “Optimal light storage with full pulse-shape control,” Phys. Rev. A **78**, 021802 (2008).