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Entangled coherent states created by mixing squeezed vacuum and coherent light: supplementary material

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This document provides supplementary information to "Entangled coherent states created by mixing squeezed vacuum and coherent light," https://doi.org/10.1364/OPTICA.6.000753. An analysis and measurements of the purity of ECS is provided, to show that states generated in our setup are highly pure entangled states, rather than mixed states. Furthermore, We have proven that our method of realizing ECS by feeding a beam splitter with pure CS is incompati-ble with a mixed superposition of NOON states with different photon numbers. To substantiate the claim that our scheme does approximate ECS, we have also added a weak amplitude descrip-tion of SV and CS mixing in Fock basis.

1. PURITY OF ECS

In this section, entanglement of ECS is examined through their purity for quantum states having corner photon statistics.

A. ECS purity through two-photon interferences

We generalize ECS to mixed states using a parameter that we introduce for the amount of purity of ECS, and show that this parameter corresponds to the visibility of two-photon interferences of the measured states. A highly pure state having corner photon statistics therefore demonstrates entanglement for any finite amplitude. We start by defining the (mixed) ECS state, $\hat{\rho}$,

$$\begin{aligned} \hat{\rho} &= \mathcal{N}^2_{\alpha,v}\left(\hat{\rho}_1+\hat{\rho}_2+v\hat{\rho}_{coh}\right), \\ \hat{\rho}_1 &= |\alpha,0\rangle\langle\alpha,0|, \\ \hat{\rho}_2 &= |0,\alpha\rangle\langle 0,\alpha|, \\ \hat{\rho}_{coh} &= |\alpha,0\rangle\langle 0,\alpha| + |0,\alpha\rangle\langle\alpha,0|, \end{aligned} \tag{S1}$$

where v is a parameter controls the purity of the state, and $\mathcal{N}_{\alpha,v} = 1/\sqrt{2\left(1+ve^{-|\alpha|^2}\right)}$. For $v=1$, the state in Eq. S1 is pure, and for $v=0$, the state is a mixed state of a coherent state in the first mode (and vacuum in the second) with a coherent state in the second mode (and vacuum in the first). The first two terms in the density matrix (Eq. S1) are the diagonal terms and represent the two-mode photon-number amplitudes. The absolute value of these quantities is verified experimentally in the main text. We only assume here that there is a phase relation for each single mode separately (which will be proved in the next section). According to our measurements and following this assumption, the density matrix in Eq. S1 represents the most general ECS.

Next, we find v by means of two-photon interferences. We first reduce the density matrix to two-photon subspace, from which the two-photon interference pattern is calculated as a function of v. In the two-photon subspace, the (unnormalized) density matrices in Fock basis are reduced as follows:

$$\begin{aligned} \hat{\rho}_1 &\to |2,0\rangle\langle 2,0|, \\ \hat{\rho}_2 &\to |0,2\rangle\langle 0,2|, \\ \hat{\rho}_{coh} &\to |2,0\rangle\langle 0,2| + |0,2\rangle\langle 2,0|, \end{aligned} \tag{S2}$$

and then,

$$\begin{aligned} \hat{\rho}_{2ph} = \frac{N^2_{\alpha,v}|\alpha|^4}{2}&\left[|2,0\rangle\langle 2,0| + |0,2\rangle\langle 0,2|\right. \\ &\left.+v\left(|2,0\rangle\langle 0,2| + |0,2\rangle\langle 2,0|\right)\right]. \end{aligned} \tag{S3}$$

The two modes are then interfered by adding a phase shift φ to one of the modes, and then mixing these modes together on a 50/50 beam-splitter. This operation is represented by the following transformation of the two-photon states:

$$|0,2\rangle \rightarrow \frac{1}{\sqrt{2}}(2|1,1\rangle + |2,0\rangle + |0,2\rangle)\,, \tag{S4}$$

$$|2,0\rangle \rightarrow \frac{e^{2i\varphi}}{\sqrt{2}}(-2|1,1\rangle + |2,0\rangle + |0,2\rangle)\,. \tag{S5}$$

Finally, we calculate the coincidence probability $P_{1,1}$, i.e. a correlation of two photons - one in either two exit ports of the beam-splitter, by projecting on the $|1,1\rangle$ state,

$$P_{1,1}(\varphi) = 2N^2_{\alpha,v}|\alpha|^4\,(1 - v\cos\varphi)\,. \tag{S6}$$

From Eq. S6, it is seen that the visibility of the two photon interference in the coincidence is the ECS purity v.

To summarize, we defined a parameter, v, as a figure of merit for the purity of the states. We showed that v can be experimentally measured by the visibility of two-photon interferences, and is measured in our setup to be $v = 0.91 \pm 0.02$ (with 95% confidence level, see Fig. S1). Thus, our measurements corresponds with generating highly pure ECS, rather than mixed unentangled states.

B. Coherence across Fock subspaces through input states purity

In the previous subsection we showed that the two-photon interferences are indication of the purity of the interfered state. Here, we show that the measured states must have coherences across its Fock subspaces, if they result from a beam splitter transformation having pure coherent states as one of their inputs. This is done through phase diffusion of ECS, i.e. generalizing of pure ECS to mixed states having a probability distribution of ECS with different phases.

Let us start by taking a simple example of a mixed state of equal probability for two ECS with opposite phases (see also Eq. 1, main text):

$$\hat{\rho}_m = \frac{1}{2}\left(|\psi^{\alpha}_{ECS}\rangle\langle\psi^{\alpha}_{ECS}| + |\psi^{-\alpha}_{ECS}\rangle\langle\psi^{-\alpha}_{ECS}|\right)\,. \tag{S7}$$

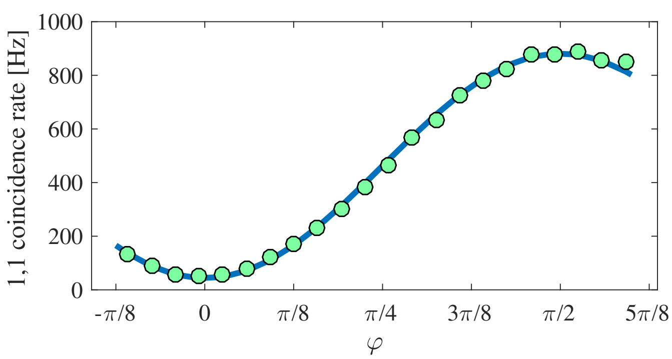

Fig. S1. Measurement of ECS purity. 1,1 coincidences rates following a beam-splitter transformation are measured for ECS in our setup, where one of its modes is phase shifted by φ (circles). The measured coincidence rate are fitted with $f_{rep} * P_{1,1}(\varphi)$ (Eq. S6) (solid line), where $f_{rep} = 80MHz$, which results with a purity value of $v = 0.91 \pm 0.02$ (with 95% confidence level).

Similarly to Eq. S1, we can rewrite Eq. S7,

$$\begin{aligned} \hat{\hat{\rho}}_m &= \mathcal{N}^2_{\alpha}\left(\hat{\hat{\rho}}_{m,1} + \hat{\hat{\rho}}_{m,2} + \hat{\hat{\rho}}_{m,coh}\right)\,, \\ \hat{\hat{\rho}}_{m,1} &= \frac{1}{2}\Big(|\alpha,0\rangle\langle\alpha,0| + |-\alpha,0\rangle\langle-\alpha,0|\Big)\,, \\ \hat{\hat{\rho}}_{m,2} &= \frac{1}{2}\Big(|0,\alpha\rangle\langle 0,\alpha| + |0,-\alpha\rangle\langle 0,-\alpha|\Big)\,, \\ \hat{\hat{\rho}}_{m,coh} &= \frac{1}{2}\Big(|\alpha,0\rangle\langle 0,\alpha| + |0,\alpha\rangle\langle\alpha,0| \\ &\quad + |-\alpha,0\rangle\langle 0,-\alpha| + |0,-\alpha\rangle\langle-\alpha,0|\Big)\,, \end{aligned} \tag{S8}$$

where $\hat{\hat{\rho}}_{m,1}$ and $\hat{\hat{\rho}}_{m,2}$ correspond to the same photon-number corner distribution of an ECS (as measured in Fig. 5, main text), while the coherence of $\hat{\hat{\rho}}_{m,coh}$ is different than that of ECS. This departure from ECS becomes clearer when $\hat{\hat{\rho}}_{m,coh}$ (Eq. S8) is rewritten in Fock basis,

$$\begin{aligned} \hat{\hat{\rho}}_{m,coh} = e^{-|\alpha|^2}\sum_{n,m=0}^{\infty}\frac{\alpha^m\alpha^{*n}}{2\sqrt{m!\,n!}}(1+(-1)^{m+n}) \\ \times(|m,0\rangle\langle 0,n| + |0,m\rangle\langle n,0|), \end{aligned} \tag{S9}$$

where the coherences between even and odd Fock subspaces vanish, while the coherences within every photon number subspace is the same as for ECS (or NOON states). Transforming the state $\hat{\rho}_m$ (Eq. S7) backwards through a beam splitter results with the state

$$\begin{aligned} \hat{\hat{\rho}}_m \rightarrow & \frac{1}{2}\Big(|\alpha/\sqrt{2}\rangle_a|\psi^{\alpha/\sqrt{2}}_{CSS}\rangle_b\,{}_a\langle\alpha/\sqrt{2}|_b\langle\psi^{\alpha/\sqrt{2}}_{CSS}| \\ & + |-\alpha/\sqrt{2}\rangle_a|\psi^{-\alpha/\sqrt{2}}_{CSS}\rangle_b\,{}_a\langle-\alpha/\sqrt{2}|_b\langle\psi^{-\alpha/\sqrt{2}}_{CSS}|\Big), \\ = & \frac{1}{2}\Big(|\alpha/\sqrt{2}\rangle_a\langle\alpha/\sqrt{2}| + |-\alpha/\sqrt{2}\rangle_a\langle-\alpha/\sqrt{2}|\Big) \\ & \otimes|\psi^{\alpha/\sqrt{2}}_{CSS}\rangle_b\langle\psi^{\alpha/\sqrt{2}}_{CSS}| \end{aligned} \tag{S10}$$

where modes a,b are the input ports of the beam splitter. From Eq. S10 it is seen the state of Eq. S7 is generated by feeding a beam splitter with CS in a statistical mixture of two opposite phases.

We note that in our setup, CS are prepared using a pulsed mode-locked laser, where coherent pulses demonstrate nearly prefect interference visibility by means of scanning a delay of a Mach-Zehnder interferometer, within a pulse or between subsequent pulses, and therefore these CS are pure states. Therefore, our setup cannot give rise to a statistical phase mixture of CS, which following a beam splitter transformation would result in a statistical mixture of NOON (Eq. S9) or entangled coherent states (Eq. S7).

Furthermore, we can exclude a larger set of mixed ECS, or mixed NOON states, having any coherence across Fock subspace, by generalizing Eq. S7 from a sum over ECS with only two phases, to an integral over all possible phases of ECS:

$$\hat{\rho}_m = \int_{-\pi}^{\pi} C(\vartheta)|\psi^{\alpha e^{i\vartheta}}_{ECS}\rangle\langle\psi^{\alpha e^{i\vartheta}}_{ECS}|\,d\vartheta, \tag{S11}$$

where $C(\vartheta)$ is the probability distribution function of the ECS phase. This state is ruled out in our setup for any $C(\vartheta)$ other than a Dirac delta function $C(\vartheta) = \delta(\vartheta)$, i.e. pure ECS, since the states in Eq. S11 require CS and CSS in phase mixed states to be fed to the beam splitter (see also Eq. S10):

$$\hat{\rho}_m \rightarrow \int_{-\pi}^{\pi} C(\vartheta)|\frac{\alpha e^{i\vartheta}}{\sqrt{2}}\rangle_a|\psi^{\alpha e^{i\vartheta}/\sqrt{2}}_{CSS}\rangle_b\,{}_a\langle\frac{\alpha e^{i\vartheta}}{\sqrt{2}}|_b\langle\psi^{\alpha e^{i\vartheta}/\sqrt{2}}_{CSS}|\,d\vartheta. \tag{S12}$$

and as stated above, CS in our setup are pure rather than mixed states.

2. WEAK AMPLITUDE DESCRIPTION OF SV AND CS MIXING IN FOCK BASIS

In addition to the discussion of fidelity to ECS (main text), we present here a theoretical proof that mixing of a coherent state with a squeezed vacuum state is a very good approximation to an ECS, by showing that both states have approximately the same amplitudes up to three photons.

First, we review the coherent and squeezed vacuum states representation using the creation mode operators ($\hat{a}^\dagger$, $\hat{b}^\dagger$) acting on the vacuum state:

$$|\beta\rangle = e^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{(\beta\hat{a}^\dagger)^n}{n!}|0\rangle, \qquad \beta=|\beta|e^{i\phi}, \tag{S13}$$

$$|\xi\rangle = \frac{1}{\sqrt{\cosh r}}\sum_{m=0}^{\infty}\frac{(-e^{i\theta}\tanh r\hat{b}^{\dagger 2})^m}{2^m m!}|0\rangle. \tag{S14}$$

By introducing the beam-splitter transformation,

$$\begin{aligned} a^\dagger &= \frac{1}{\sqrt{2}}(c^\dagger + d^\dagger), \\ b^\dagger &= \frac{1}{\sqrt{2}}(c^\dagger - d^\dagger), \end{aligned} \tag{S15}$$

the state $|\beta_a\rangle \otimes |\xi\rangle_b$, following the beam-splitter transformation (Eq. S15) is

$$\begin{aligned} |\psi_{out}\rangle = &\frac{e^{-|\beta|^2/2}}{\sqrt{\cosh r}}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\beta^n(\hat{c}^\dagger+\hat{d}^\dagger)^n}{\sqrt{2}^n n!} \\ &\times\frac{(-e^{i\theta}\tanh r)^m(\hat{c}^{\dagger 2}+\hat{d}^{\dagger 2}-2\hat{c}^\dagger\hat{d}^\dagger)^m}{2^{2m}m!}|0\rangle_c|0\rangle_d. \end{aligned} \tag{S16}$$

Summing contributions up three photons, we apply the sum on m for $m = 0, 1$. For $m = 0$, and $n = 0, 1, 2, 3$, we get the following terms, up to a prefactor of $\frac{e^{-|\beta|^2/2}}{\sqrt{\cosh r}}$:

$$\begin{aligned} &|0\rangle_c|0\rangle_d + \frac{\beta}{\sqrt{2}}(|1\rangle_c|0\rangle_d + |0\rangle_c|1\rangle_d) \\ &\quad+\frac{\beta^2}{4}\left(\sqrt{2}|2\rangle_c|0\rangle_d + 2|1\rangle_c|1\rangle_d + \sqrt{2}|0\rangle_c|2\rangle_d\right) \\ &\quad+\frac{\beta^3}{6\sqrt{8}}\Big(\sqrt{6}|3\rangle_c|0\rangle_d + 3\sqrt{2}|2\rangle_c|1\rangle_d + 3\sqrt{2}|1\rangle_c|2\rangle_d \\ &\qquad\qquad + \sqrt{6}|0\rangle_c|3\rangle_d\Big), \end{aligned} \tag{S17}$$

and for $m = 1, n = 0, 1$, we get, up to the same $\frac{e^{-|\beta|^2/2}}{\sqrt{\cosh r}}$ prefactor, the following terms:

$$\begin{aligned} &\frac{(-e^{i\theta}\tanh r)}{4}\left(\sqrt{2}|2\rangle_c|0\rangle_d - 2|1\rangle_c|1\rangle_d + \sqrt{2}|0\rangle_c|2\rangle_d\right) \\ &\quad+\frac{(-e^{i\theta}\tanh r)}{4}\frac{\beta}{\sqrt{2}}\Big(\sqrt{6}|3\rangle_c|0\rangle_d - \sqrt{2}|2\rangle_c|1\rangle_d \\ &\qquad - \sqrt{2}|1\rangle_c|2\rangle_d + \sqrt{6}|0\rangle_c|3\rangle_d\Big). \end{aligned} \tag{S18}$$

By summing the two contributions of Eqs. S17-S18, the state of Eq. S16, up to three photons becomes:

$$\begin{aligned} |\tilde{\psi}_{out}\rangle = \; & |0\rangle_c|0\rangle_d + \frac{\beta}{\sqrt{2}}(|1\rangle_c|0\rangle_d + |0\rangle_c|1\rangle_d) + \\ & \frac{\beta^2 - e^{i\theta}\tanh r}{4}\sqrt{2}\,(|2\rangle_c|0\rangle_d + |0\rangle_c|2\rangle_d) + \\ & \frac{\beta^2 + e^{i\theta}\tanh r}{2}|1\rangle_c|1\rangle_d + \\ & \frac{(\frac{1}{3}\beta^2 - e^{i\theta}\tanh r)\beta}{4}\sqrt{3}\,(|3\rangle_c|0\rangle_d + |0\rangle_c|3\rangle_d) + \\ & \frac{(\beta^2 + e^{i\theta}\tanh r)\beta}{4}(|2\rangle_c|1\rangle_d + |1\rangle_c|2\rangle_d), \end{aligned} \tag{S19}$$

where a prefactor of $\frac{e^{-|\beta|^2/2}}{\sqrt{\cosh r}}$ has been omitted in Eq. S19. By substituting $e^{i\theta}\tanh r = -\beta^2$ in Eq. S19, $|\tilde{\psi}_{out}\rangle$ becomes:

$$\begin{aligned} |\tilde{\psi}_{out}\rangle = \; & |0\rangle_c|0\rangle_d + \frac{\beta}{\sqrt{2}}(|1\rangle_c|0\rangle_d + |0\rangle_c|1\rangle_d) + \\ & \frac{\beta^2}{\sqrt{2}}(|2\rangle_c|0\rangle_d + |0\rangle_c|2\rangle_d) + \\ & \frac{\beta^3}{\sqrt{3}}(|3\rangle_c|0\rangle_d + |0\rangle_c|3\rangle_d), \end{aligned} \tag{S20}$$

where in Eq. S20, we retrieve the corner-like photon number distribution, for low photon number. We note that the condition $e^{i\theta}\tanh r = -\beta^2$, that was applied on Eq. S20 is similar to the condition found in the main text; $\tanh r = |\beta|^2 = |\alpha|^2/2$, instead of $\sinh 2r = |\alpha|^2$ (main text, Eq.9), which is approximately the same condition for small r. Under this approximation, the terms of the corner-like state, for $\alpha = \sqrt{2}\beta$ match those of ECS, up to three photons [1]:

$$\begin{aligned} |\tilde{\psi}_{out}\rangle = \; & \frac{1}{2}\Big(2|0\rangle_c|0\rangle_d + \alpha\,(|1\rangle_c|0\rangle_d + |0\rangle_c|1\rangle_d) + \\ & \frac{\alpha^2}{\sqrt{2}}(|2\rangle_c|0\rangle_d + |0\rangle_c|2\rangle_d) + \\ & \frac{\alpha^3}{\sqrt{6}}(|3\rangle_c|0\rangle_d + |0\rangle_c|3\rangle_d)\Big), \end{aligned} \tag{S21}$$

while the omitted prefactor in $|\tilde{\psi}_{out}\rangle$, $\frac{e^{-|\beta|^2/2}}{\sqrt{\cosh r}}$, is also approximated by $2\mathcal{N}_\alpha exp(-|\alpha|^2/2)$ (see Eq. 1, main text) for small r .

REFERENCES

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