

# Temporal soliton and optical frequency comb generation in Brillouin laser cavity: supplementary material

YALI HUANG<sup>1,†</sup>, QING LI<sup>1,†</sup>, JUNYUAN HAN<sup>2,3,†</sup>, ZHIXU JIA<sup>1</sup>, YONGSEN YU<sup>1</sup>, YUEDE YANG<sup>2,3,\*</sup>, JINLONG XIAO<sup>2,3</sup>, JILIANG WU<sup>2,3</sup>, DAMING ZHANG<sup>1</sup>, YONGZHEN HUANG<sup>2,3,\*</sup>, WEIPING QIN<sup>1</sup>, AND GUANSHI QIN<sup>1,\*</sup>

<sup>1</sup>State Key Laboratory of Integrated Optoelectronics, College of Electronic Science and Engineering, Jilin University, 2699 Qianjin Street, Changchun 130012, China

<sup>2</sup>State Key Laboratory of Integrated Optoelectronics, Institute of Semiconductors, Chinese Academy of Sciences, Beijing 100083, China

<sup>3</sup>Center of Materials Sciences and Optoelectrics Engineering, University of Chinese Academy of Sciences, Beijing 100049, China

\*Corresponding author: [yyd@semi.ac.cn](mailto:yyd@semi.ac.cn), [yzhuang@semi.ac.cn](mailto:yzhuang@semi.ac.cn), [qings@jlu.edu.cn](mailto:qings@jlu.edu.cn)

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## 1. The group velocity dispersion of the HNLF

The HNLF used in our experiments was made by Yangtze Optical Fiber and Cable Company Ltd. (YOFC). The group velocity dispersion profile of the HNLF is shown in Fig. S1,  $\beta_2$  and  $\beta_3$  at the wavelength of pump wave (1550 nm) are  $-0.5 \text{ ps}^2 \text{ km}^{-1}$  and  $0.04 \text{ ps}^3 \text{ km}^{-1}$ , respectively.

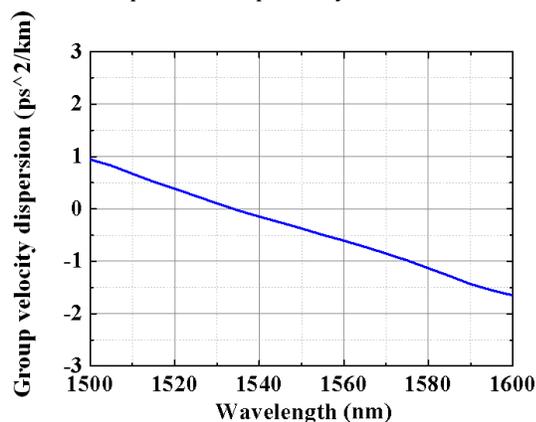


Fig. S1. Group velocity dispersion profile of the HNLF

## 2. Multiple wavelength CW lasers and generated intra-cavity multiple wavelength Brillouin lasers

The pump source we used for temporal soliton and optical frequency comb generation was multiple wavelength CW lasers system. Two CW single frequency laser beams at the C band (line widths  $\sim 150 \text{ kHz}$ , two external cavity semiconductor lasers) with a frequency separation  $f$  were amplified with a two-stage  $\text{Er}^{3+}$ -doped fiber amplifier (EDFA) and launched into a 500 m highly nonlinear fiber for achieving multiple-wavelength narrow-linewidth CW lasers with an equal frequency separation  $f$  via cascaded FWM. The

generated multiple-wavelength narrow-linewidth CW lasers were launched into the ring cavity by an optical circulator for generating multiple-wavelength Brillouin lasers inside the cavity, which were used as the internal driving light for temporal cavity soliton and optical frequency comb generation. Spectrum of the multiple wavelength CW lasers and the intra-cavity multiple wavelength Brillouin lasers in frequency separation of 80 GHz are shown in Fig. S2.

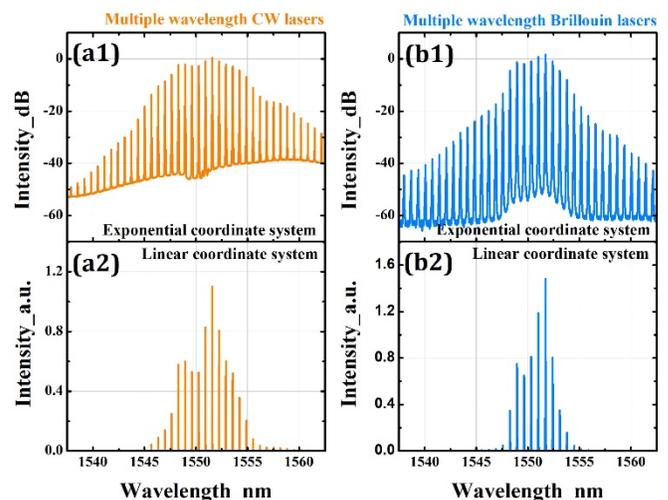


Fig. S2. Spectrum of the multiple wavelength CW lasers in frequency separation of 80 GHz as the external pump laser (a1) in exponential and (a2) linear coordinate system. Spectrum of the intra-cavity multiple wavelength Brillouin lasers and frequency comb in frequency separation of 80 GHz as the internal pump laser (b1) in exponential and (b2) linear coordinate system when the pump power is 200 mW.

### 3. RF spectra of the Brillouin laser and cavity longitudinal modes

The linewidth of the generated Brillouin laser was measured by using delayed self heterodyne method [1]. The delayed self-heterodyne beating radio frequency (RF) spectra of the Brillouin laser and cavity longitudinal modes are shown in Fig. S3. The cavity longitudinal modes are spaced in frequency separation of 622 kHz, one of the cavity modes are amplified as the Brillouin laser. The spectrum of the Brillouin laser is fitted by Lorentzian curve, the Lorentz fit linewidth of which is 4.8 kHz, indicating that the linewidth of the resonance mode is narrower than or similar with 4.8 kHz.

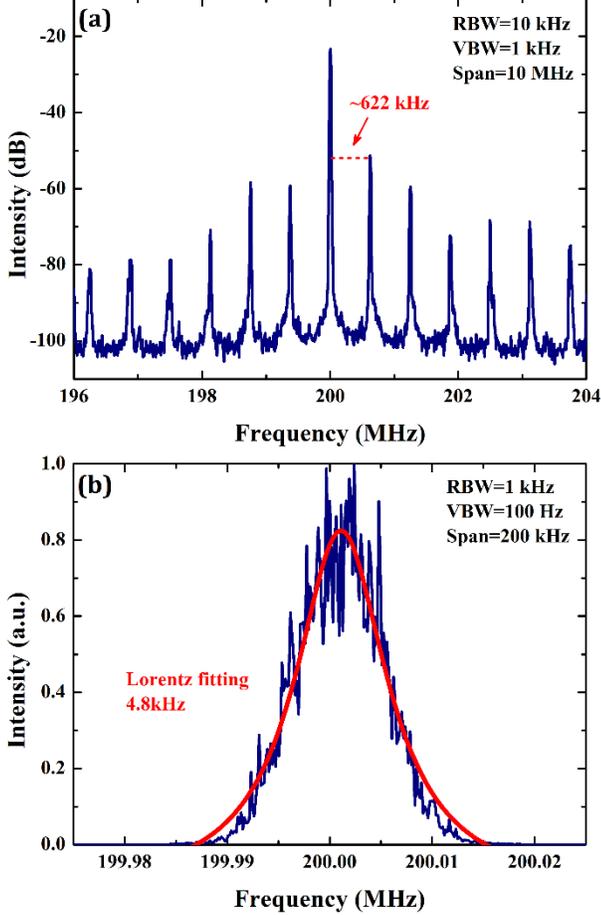


Fig. S3. The delayed self-heterodyne beating RF spectra of (a) the Brillouin laser and cavity longitudinal modes (FSR: 622 kHz) (b) Brillouin laser and its Lorentz fitting. RBW: resolution bandwidth; VBW: video bandwidth.

### 4. Numerical simulations of temporal soliton and optical frequency comb generation

To interpret our experimental results (shown in Fig. 3), we performed numerical simulations by solving the nonlinear coupled-mode equations [2].

$$\frac{\partial A_\mu}{\partial t} = -\frac{\kappa}{2} A_\mu + \delta_{\mu 0} \sqrt{\eta \kappa} s_{in} e^{-i(\omega_p - \omega_0)t} \quad (S1)$$

$$+ i g \sum_{\mu', \mu'', \mu'''} A_{\mu'} A_{\mu''} A_{\mu'''}^* e^{-i(\omega_{\mu'} + \omega_{\mu''} - \omega_{\mu'''} - \omega_\mu)t}$$

$$s_{out} = s_{in} - \sqrt{\eta \kappa} \sum A_\mu e^{-i(\omega_\mu - \omega_p)t} \quad (S2)$$

where  $A_\mu$  is the evolution of the mode amplitudes,  $\mu$  is the mode with index which is defined relative to pump mode  $\mu = 0$ , resonance frequency is  $\omega_\mu = \omega_0 + D_1 \mu + D_2 \mu^2 / 2 + D_3 \mu^3 / 6 + \dots$  ( $D_1 = 2\pi / T_R$ ,  $D_2$  and  $D_3$  corresponding to FSR, second and third order dispersion, fourth and higher order dispersion terms may be introduced in analogous manner),  $t$  is time parameter,  $\kappa = \kappa_0 + \kappa_{ext}$  is the cavity decay rate as the sum of intrinsic decay rate  $\kappa_0$  and coupling rate to the waveguide  $\kappa_{ext}$ ,  $\eta = \kappa_{ext} / \kappa$  is the coupling efficiency,  $\delta_{\mu 0}$  is the Kronecker delta,  $\omega_p$  is frequency of pump laser, and  $|s_{in, out}| = \sqrt{P_{in, out} / \hbar \omega_0}$  is the amplitudes of the pump and output powers. In the simulation, the nonlinear coupling coefficient

$$g = \frac{\hbar \omega_0^2 c n_2}{n_0^2 V_{eff}} \quad (S3)$$

describes the cubic Kerr-nonlinearity of the system with the refractive index  $n_0$ , nonlinear refractive index  $n_2$ , the effective cavity nonlinear volume  $V_{eff} = A_{eff} L$  (with effective nonlinear mode-area  $A_{eff}$  and circumference of the cavity  $L$ ), the speed of light  $c$  and the Planck constant  $\hbar$ . The summation includes all  $\mu'$ ,  $\mu''$ ,  $\mu'''$  respecting the relation  $\mu = \mu' + \mu'' - \mu'''$ .

$$\omega_n = \omega_0 + 3 \frac{|\beta_2|}{\beta_3} \quad (S4)$$

Equation (S4) is the phase match equation [3] where  $\omega_n$  and  $\omega_0$  are the carrier angular frequency of the dispersive wave and the pump wave, respectively.  $\beta_2$  and  $\beta_3$  at the wavelength of pump wave (1550 nm) are  $-0.5 \text{ ps}^2 \text{ km}^{-1}$  and  $0.04 \text{ ps}^3 \text{ km}^{-1}$  respectively, as shown in Fig. S1. The absolute value of the frequency shift  $F((\omega_n - \omega_0) / 2\pi)$  which is equivalent to  $3|\beta_2| / 2\pi\beta_3$  from the equation (1) is calculated out to be 5.96 THz (corresponding to the wavelength of 1504.5 nm). The value is agreed with the experimental results and numerical results as shown in Fig. 3(b) and Fig. 4(b), the frequency and wavelength location of the dispersive wave.

### 5. Long term stability of the pulse profiles of the cavity soliton

In order to confirm the long term operation stability of the temporal cavity solitons, we measured the pulse profiles of the temporal cavity solitons every 10 min for half an hour. The measured pulse profiles are shown in Fig. S4, which shows that the temporal cavity solitons are stable for more than half an hour.

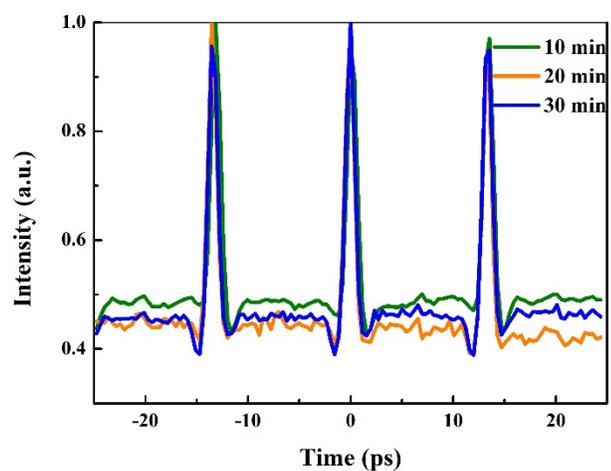


Fig. S4. Long term stability of the cavity soliton pulse profiles tested in half an hour.

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## REFERENCES

- [1] T. Okoshi, K. Kikuchi, and A. Nakayama, "Novel method for high resolution measurement of laser output spectrum," *Electron. Lett.* **16**, 630–631 (1980).
- [2] T. Herr, V. Brasch, J. D. Jost, C. Y. Wang, N. M. Kondratiev, M. L. Gorodetsky, and T. J. Kippenberg, "Temporal solitons in optical microresonators," *Nature Photonics* **8**, 145–152, (2014).
- [3] M. Erkintalo, Y. Q. Xu, S. G. Murdoch, J. M. Dudley, and G. Genty, "Cascaded phase matching and nonlinear symmetry breaking in fiber frequency combs," *Phys. Rev. Lett.* **109**, 223904 (2012).