Supplemental Document

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### SI. Calculation of the reflection matrix

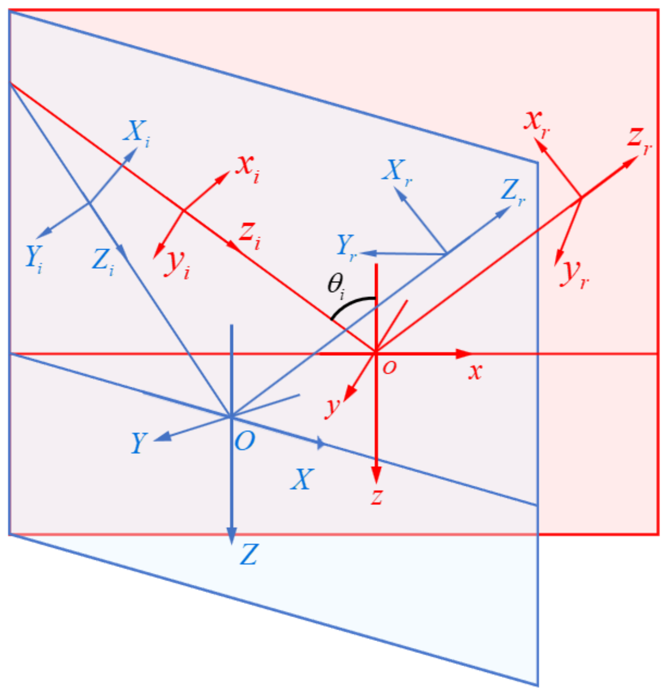


Fig.S1 Scheme of the reflection of the central wave and the local wave vector at the air-glass interface.

In order to convert the frame $(x_i, y_i, z_i)$ to the frame $(X_i, Y_i, Z_i)$, these three steps should be performed (Fig.S1). Firstly, we transform the coordinate system $(x_i, y_i, z_i)$ around the $y$ axis at the incident angle $\theta_i$ to the frame $(x, y, z)$. The first changing matrix is given by:

$$M_{x_i y_i z_i \to xyz} = \begin{bmatrix} \cos(-\theta_i) & 0 & \sin(-\theta_i) \\ 0 & 1 & 0 \\ -\sin(-\theta_i) & 0 & \cos(-\theta_i) \end{bmatrix}. \tag{S1}$$

Secondly, we need to transform the frame $(x, y, z)$ around the $z$ axis by an angle $\frac{k_{iy}}{k_0 \sin\theta_i}$ to the frame $XYZ$, and the second matrix can be written as the follow:

$$M_{xyz \to XYZ} = \begin{bmatrix} 1 & \frac{k_y}{k_0 \sin\theta_i} & 0 \\ -\frac{k_y}{k_0 \sin\theta_i} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{S2}$$

At the end step, we transform the frame $XYZ$ around the $y$ axis by an angle $-\theta_i$ to the frame $X_i Y_i Z_i$, and the third matrix is given by:

$$M_{XYZ \to X_i Y_i Z_i} = \begin{bmatrix} \cos\theta_i & 0 & \sin\theta_i \\ 0 & 1 & 0 \\ -\sin\theta_i & 0 & \cos\theta_i \end{bmatrix}.$$

(S3)

In summary, the matrix of the whole transformation can be written as the following:

$$\begin{aligned} M_{x_i y_i z_i \to X_i Y_i Z_i} &= M_{XYZ \to X_i Y_i Z_i} M_{xyz \to XYZ} M_{x_i y_i z_i \to xyz} \\ &= \begin{bmatrix} \cos\theta_i & 0 & \sin\theta_i \\ 0 & 1 & 0 \\ -\sin\theta_i & 0 & \cos\theta_i \end{bmatrix} \begin{bmatrix} 1 & \frac{k_y}{k_0 \sin\theta_i} & 0 \\ -\frac{k_y}{k_0 \sin\theta_i} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(-\theta_i) & 0 & \sin(-\theta_i) \\ 0 & 1 & 0 \\ -\sin(-\theta_i) & 0 & \cos(-\theta_i) \end{bmatrix} \\ &= \begin{bmatrix} 1 & \frac{k_{iy} \cot\theta_i}{k_0} & 0 \\ -\frac{k_{iy} \cot\theta_i}{k_0} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{S4}$$

The two-dimensional form of this matrix can be written as:

$$M_{x_iy_iz_i\to X_iY_iZ_i}=\begin{bmatrix} 1 & \dfrac{k_y\cot\theta_i}{k_0} \\ -\dfrac{k_y\cot\theta_i}{k_0} & 1 \end{bmatrix}. \tag{S5}$$

Following the procedure above, we can transform the frame $x_ry_rz_r$ to the frame $X_rY_rZ_r$ :

$$M_{X_rY_rZ_r\to x_ry_rz_r}=\begin{bmatrix} 1 & \dfrac{k_y\cot\theta_i}{k_0} \\ -\dfrac{k_y\cot\theta_i}{k_0} & 1 \end{bmatrix}. \tag{S6}$$

As we know, for any arbitrary wave vector, the reflected field is determined by $E^{p,s}_{X_rY_rZ_r}=r_{p,s}E^{p,s}_{X_iY_iZ_i}$ , $r_p$ and $r_s$ refer to the Fresnel reflection coefficient of the p and s wave, respectively. Therefore, the reflection matrix is as the follows:

$$\begin{aligned} M_R &= M_{X_rY_rZ_r\to x_ry_rz_r}\begin{bmatrix} r_p & 0 \\ 0 & r_s \end{bmatrix} M_{x_iy_iz_i\to X_iY_iZ_i} \\ &= \begin{bmatrix} 0 & \dfrac{k_y\cot\theta_i}{k_0} \\ -\dfrac{k_y\cot\theta_i}{k_0} & 0 \end{bmatrix}\cdot\begin{bmatrix} r_p & 0 \\ 0 & r_s \end{bmatrix}\cdot\begin{bmatrix} 0 & \dfrac{k_y\cot\theta_i}{k_0} \\ -\dfrac{k_y\cot\theta_i}{k_0} & 0 \end{bmatrix}. \qquad \text{(S7)} \\ &= \begin{bmatrix} r_p & \dfrac{k_y\left(r_P+r_s\right)\cot\theta_i}{k_0} \\ -\dfrac{k_y\left(r_p+r_s\right)\cot\theta_i}{k_0} & r_s \end{bmatrix} \end{aligned}$$

**SII. Calculation of spatial spectral transfer function**

When the input field $\tilde{E}_{in}$ passes through the GLP1, we can get the field with $\alpha$ linear polarization with $x$ axis:

$$\tilde{\mathbf{E}}_{in}=\tilde{E}_{in}\begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}. \tag{S8}$$

After the light beam passing through the QWP, whose optical axis is 0° with $x$ direction, and the Jones matrix of the QWP can be written as:

$$\langle QWP| = \begin{bmatrix} \exp\left(-\frac{\pi}{4}i\right) & 0 \\ 0 & \exp\left(\frac{\pi}{4}i\right) \end{bmatrix}. \tag{S9}$$

Then the light beam reflected at the glass-air interface, the angular spectrum can be obtained by the relation $\tilde{\mathbf{E}}_r = R_M \langle QWP|\tilde{\mathbf{E}}_{in}$. The reflection matrix can be written as:

$$R_M = \begin{bmatrix} r_p & k\Delta \\ -k\Delta & r_s \end{bmatrix}, \tag{S10}$$

where $r_p$ and $r_s$ are the Fresnel reflection coefficients of p and s waves, $\Delta = (r_p + r_s)\cot\theta_i / k_0$, $k_0 = 2\pi/\lambda$ is the wave vector in the vacuum, $\theta_i$ is incident angle. Based on the boundary condition, the angular spectrum of reflected light field can be written as

$$\begin{aligned} \tilde{\mathbf{E}}_r = \tilde{E}_{in} \Bigg\{ & \left[ \exp\left(-\frac{\pi}{4}i\right) r_p \cos\alpha + \exp\left(\frac{\pi}{4}i\right) \Delta \sin\alpha k_y \right] e_x \\ & + \left[ \exp\left(\frac{\pi}{4}i\right) r_s \sin\alpha - \exp\left(-\frac{\pi}{4}i\right) \Delta \cos\alpha k_y \right] e_y \Bigg\} \end{aligned}. \tag{S11}$$

Here, $e_x = \frac{1}{\sqrt{2}}(e_- + e_+)$, $e_y = \frac{i}{\sqrt{2}}(e_- - e_+)$. After substituting them into the (S4), the angular spectrum of reflected light field can also be written as

$$\begin{aligned} \tilde{\mathbf{E}}_r = \frac{\tilde{E}_{in}}{\sqrt{2}} \Bigg\{ & \exp\left(-\frac{\pi}{4}i\right) \cos\alpha \left(r_p - i\Delta k_y\right) e_- + \exp\left(-\frac{\pi}{4}i\right) \cos\alpha \left(r_p + i\Delta k_y\right) e_+ \\ & + i \exp\left(\frac{\pi}{4}i\right) \sin\alpha \left(r_s - i\Delta k_y\right) e_- - i \exp\left(\frac{\pi}{4}i\right) \sin\alpha \left(r_s + i\Delta k_y\right) e_+ \Bigg\} \end{aligned}, \tag{S12}$$

according to the Taylor series expansion, $r_p$ and $r_s$ can be expanded as follows

$$\begin{aligned} r_p(k_x) &= r_p(k_x = 0) + k_x \left[ \frac{\partial r_p(k_x)}{\partial k_x} \right]_{k_x=0}, \\ r_s(k_x) &= r_s(k_x = 0) + k_x \left[ \frac{\partial r_s(k_x)}{\partial k_x} \right]_{k_x=0}. \end{aligned} \tag{S13}$$

Then the angular spectrum of reflected light field can be rewritten as

$$\tilde{\mathbf{E}}_r = \exp\left(-\frac{\pi}{4}i\right)\frac{\sin\alpha\tilde{E}_{in}r_s}{\sqrt{2}}\left\{\left[1+\left(\frac{r_p}{r_s\tan\alpha}\Delta_H+\Delta_V\right)k_x+i\left(\delta_H+\delta_V\right)k_y\right]e_+ \right.$$
$$\left. -\left[1+\left(-\frac{r_p}{r_s\tan\alpha}\Delta_H+\Delta_V\right)k_x+i\left(\delta_H-\delta_V\right)k_y\right]e_-\right\}, \quad \text{(S14)}$$

where $\Delta_H = \frac{\partial lnr_p}{k_0\partial\theta_i}$, $\Delta_V = \frac{\partial lnr_s}{k_0\partial\theta_i}$, $\delta_H = \frac{\Delta}{r_s\tan\alpha}$, $\delta_V = \frac{\Delta}{r_s}$. Then we set $\Delta x = \frac{r_p}{r_s\tan\alpha}\Delta H$, $\Delta y = \delta_V$, and the reflected field can be written as

$$\tilde{\mathbf{E}}_r \approx \exp\left(-\frac{\pi}{4}i\right)\frac{\sin\alpha\tilde{E}_{in}r_s}{\sqrt{2}}\left[\exp\left(\Delta xk_x+i\Delta yk_y\right)e_+-\exp\left(-\Delta xk_x-i\Delta yk_y\right)e_-\right]. \quad \text{(S14)}$$

Here, we have introduced the approximation: $1+\Delta xk_x+\Delta yk_y \approx \exp\left(\Delta xk_x+\Delta yk_y\right)$.

Then the reflected field passes through second polarizer whose polarization axis is chosen as $0°$. Therefore, the output field in the whole differentiator system can be acquired as

$$\tilde{E}_{out} = \exp\left(-\frac{\pi}{4}i\right)\frac{\sin\alpha\tilde{E}_{in}r_s}{\sqrt{2}}\left[\exp\left(\Delta xk_x+i\Delta yk_y\right)-\exp\left(-\Delta xk_x-i\Delta yk_y\right)\right]. \quad \text{(S15)}$$

In order to get the homogeneous two-dimensional isotropic edges in the subsequent experiments, we need to ensure that $\Delta x = \Delta y$ , and then we can obtain the $\alpha = 67.1°$.

According to the expression of the spatial transfer function

$$H\left(k_x,k_y\right) = \frac{\tilde{E}_{out}\left(k_x,k_y\right)}{\tilde{E}_{in}\left(k_x,k_y\right)}. \quad \text{(S16)}$$

After substituting Eq. (S9) into Eq. (S10), we obtain

$$\begin{aligned} H\left(k_x,k_y\right) &\propto i\tilde{E}_{in}r_s\sin\left(-i\Delta xk_x+\Delta yk_y\right) \\ &\simeq \Delta xk_x+i\Delta yk_y \end{aligned}. \quad \text{(S17)}$$

**SIII. Calculation the theoretical bandwidth of the differentiator**

In the Fig.S2, we chose three different beam widths in the x and y axis, respectively. In our experimental system, the beam width is 0.02mm, which is red lines shown in the Fig.S2. We find that the gain of the system become larger when the beam width is smaller.

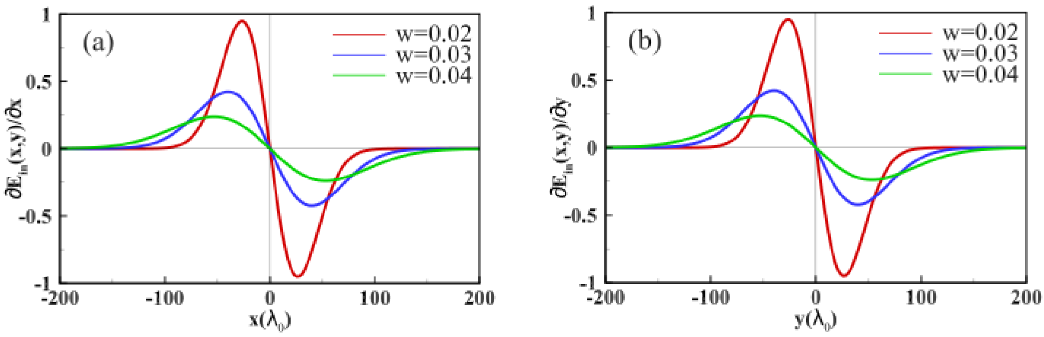


Fig.S2. Calculation results for a signal wave, including a Gaussian field with different beam widths incident on the prism. (a). The first-order derivative of the input field in the x direction. (b). The first-order derivative of the input field in the y direction. The ordinate value is normalized.

## SIV. Calculation the theoretical of the output field

The output field in position space can be obtained by the Fourier transform: [S1]

$$E_{out} = \iint \tilde{E}_{out} \exp\left[ i\left( k_x x + k_y y \right) \right] dk_x dk_y . \tag{S18}$$

After substituting Eq. (S8) into Eq. (S11), the output field in position space can be written as

$$\begin{aligned} E_{out}\left(x, y\right) &\simeq r_s \left[ E_{in}\left(x + \Delta x, y + i\Delta y\right) - E_{in}\left(x - \Delta x, y - i\Delta y\right) \right] \\ &= \Delta x \frac{\partial E_{in}(x, y)}{\partial x} + i\Delta y \frac{\partial E_{in}(x, y)}{\partial y} \end{aligned} . \tag{S19}$$

Therefore, the output field is approximately proportional to the two-dimensional spatial differentiation of the input field.

**Supplementary References**


[S1] J. W. Goodman, Introduction to Fourier Optics (McGraw-Hill, 1968).