Supplemental Document

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Nonlinear color space coded by additive digital pulses: supplement

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1. Encoding algorithm

A coordinate (X_i, Y_i, Z_i) in the CIE 1931 *XYZ* tristimulus space can be transformed from the CIE 1931 *xy*Y color space following [1]:

$$\begin{cases} X_i = x_i \frac{Y_i}{y_i} \\ Y_i = \mathrm{L}_i \\ Z_i = z_i \frac{Y_i}{y_i} \end{cases} \tag{S1}$$

where x_i , y_i and L_i represents the chromaticity *x*, *y* and luminosity of *i*th light source, respectively.

Considering the duty cycle of driving pulse of *i*th light source is D_i, the luminosity is modulated as $\mathrm{L}_i D_i$. Further define $R_i = \mathrm{L}_i / y_i$, the equations (S1) can be rewritten as:

$$\begin{cases} X_i = x_i R_i D_i \\ Y_i = y_i R_i D_i \\ Z_i = z_i R_i D_i \end{cases} \tag{S2}$$

The coordinate (X_m, Y_m, Z_m) of additive mixed light in the CIE 1931 *XYZ* tristimulus space can be obtained as:

$$\begin{cases} X_m = \sum X_i \\ Y_m = \sum Y_i \\ Z_m = \sum Z_i \end{cases} \tag{S3}$$

Transform the coordinate (X_m, Y_m, Z_m) to the CIE 1931 *xyz* color space:

$$\begin{cases} x_m = \frac{X_m}{X_m + Y_m + Z_m} \\ y_m = \frac{Y_m}{X_m + Y_m + Z_m} \\ z_m = \frac{Z_m}{X_m + Y_m + Z_m} \end{cases} \tag{S4}$$

Substituting (S2) and (S3) into (S4), chromaticity *x* of the mixed light can be got:

$$x_m = \frac{\sum x_i R_i D_i}{\sum x_i R_i D_i + \sum y_i R_i D_i + \sum z_i R_i D_i} = \frac{\sum x_i R_i D_i}{\sum R_i D_i} \tag{S5}$$

Similarly, one can obtain chromaticity *y*:

$$y_m = \frac{\sum y_i R_i D_i}{\sum R_i D_i} \tag{S6}$$

Equations (S5) and (S6) are equivalent to the equations ① and ② in the main text, respectively.

2. Decoding algorithm

Rewrite the equations (①~③) in the main text as:

$$\begin{cases} x_m = \dfrac{\sum x_i R_i D_i}{\sum R_i D_i} & \text{(S7)} \\ y_m = \dfrac{\sum y_i R_i D_i}{\sum R_i D_i} & \text{(S8)} \\ \mathrm{L}_m = \sum \mathrm{L}_i D_i = \sum y_i R_i D_i & \text{(S9)} \end{cases}$$

where i=1,2,3 for the light mixing with three primary lights.

Defining $R_m = \sum R_i D_i$, then equations (S7~S9) are equivalent to

$$\begin{cases} R_m = R_1 D_1 + R_2 D_2 + R_3 D_3 & \text{(S10)} \\ x_m R_m = R_1 D_1 x_1 + R_2 D_2 x_2 + R_3 D_3 x_3 & \text{(S11)} \\ y_m R_m = R_1 D_1 y_1 + R_2 D_2 y_2 + R_3 D_3 y_3 & \text{(S12)} \end{cases}$$

since (S7) = (S11)/(S10), (S8) = (S12)/(S10) and (S9) = (S12).

From (S10), one obtains:

$$R_1 D_1 = R_m - R_2 D_2 - R_3 D_3 \tag{S13}$$

Substituting (S13) into (S11), one obtains:

$$R_2 D_2 = \frac{R_m(x_m - x_1) - R_3 D_3 (x_3 - x_1)}{x_2 - x_1} \tag{S14}$$

Substituting (S13) into (S12), one obtains:

$$(y_m - y_1) R_m = R_2 D_2 (y_2 - y_1) + R_3 D_3 (y_3 - y_1) \tag{S15}$$

Substituting (S14) into (S15), one obtains D_3. D_1 and D_2 can be obtained by exchanging the subscripts.

3. Nonlinear gamut boundaries

Let $i = 1, k = 2, j = 3$ and $D_1 = 1$, equation ④ in the main text becomes:

$$\frac{(x_3 - x_2)(y_m - y_2) - (y_3 - y_2)(x_m - x_2)}{(x_3 - x_2)(y_1 - y_2) - (y_3 - y_2)(x_1 - x_2)} \frac{\mathrm{R}_m}{\mathrm{R}_1} = 1 \tag{S16}$$

From the equations (S9) and (S12), one gets:

$$\mathrm{R}_m = \mathrm{L}_m / y_m \tag{S17}$$

Substituting (S17) into (S16), one obtains the upper gamut boundary of light source 1:

$$\mathrm{L}_b^1(x_m, y_m) = \frac{(x_3 - x_2)(y_1 - y_2) - (y_3 - y_2)(x_1 - x_2)}{(x_3 - x_2)(y_m - y_2) - (y_3 - y_2)(x_m - x_2)} y_m \mathrm{R}_1 \tag{S18}$$

The upper physical boundaries for the other two lights can be obtained in the same way.

4. Upper physical gamut boundaries and upper gamut surfaces

We take the primary light "2" and "3" for example to show the definitions of upper physical gamut boundaries and upper gamut surfaces and their differences. As it can be seen from Fig. S1(a), the upper physical boundaries for the primary light "2" (L_b^2, green lines) and "3" (L_b^3, red lines) are calculated in accordance with the equation of ⑨ in the main text, respectively. As it can be seen, the two boundaries are overlapped each other in certain gamut. The upper gamut surfaces satisfying both boundary conditions (especially the digital constrain ⑧ in the main text) take the lower parts of physical gamut boundaries in the overlapped regions, as shown in Fig. S1(b).

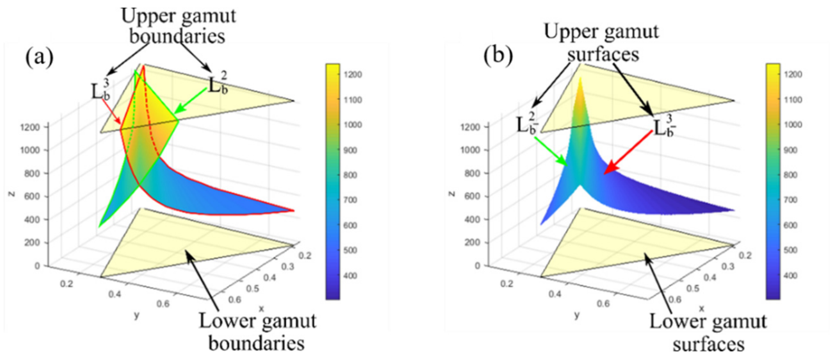

Fig. S1. Two components of 3D gamut boundaries (a) and gamut surfaces (b)

5. Codable ratio for quantifying light configuration qualities

For biprimary color mixing, two configurations of an office luminaire made of "cold" and "warm" white LEDs are considered. The equation of codable ratio (equation ⑪ in the main text) can be simplified as:

$$\delta = \frac{\int_{x_c}^{x_w} \mathrm{L}_b(x)\mathrm{d}x}{(x_w - x_c)\times(\mathrm{L}_c + \mathrm{L}_w)} \tag{S19}$$

where x_c, x_w are the chromaticity x of the "cold" and "warm" white LEDs, respectively. L_c and L_w are the luminosities of the "cold" and "warm" white LEDs, respectively. L_b is the gamut boundary in the 2D CIE *xy*Y color space. The color parameters for the primary lights are set in Table S1.

Table S1. The color parameters of two configurations of an office luminaire

LEDs	Cold white LEDs			Warm white LEDs		
Para.	*x*	*y*	Flux/lm	*x*	*y*	Flux/lm
Config. *a*	0.306	0.3187	200	0.4322	0.398	200
Config. *b*	0.306	0.3187	200	0.4322	0.398	300

2D gamut boundaries of two configurations *a* and *b* in the CIE 1931 *xy*Y color space are plotted in Fig. S2(a) and Fig. S2(b), respectively. As can be seen, the two configurations keep the same chromaticity but different luminosities of "warm" white LEDs (the "cold" white LEDs keep the same). Granted, the configuration *b* will have a relative larger realizable color space, considering a larger luminosity of "warm" white LEDs. In contrast, the codable ratios for configuration *a* and *b* are calculated as 0.6915 and 0.6851, respectively. Therefore, configuration *a* has a larger accessible space and thus a greater tunable capability.

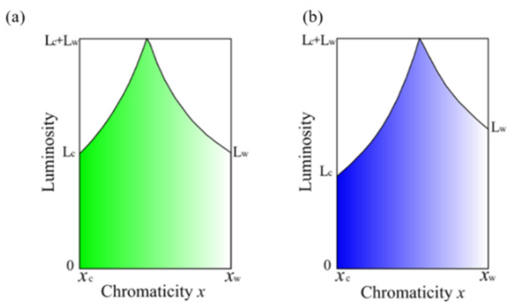

Fig. S2. 2D theoretical and accessible spaces of two configurations for biprimary mixing luminaire

For triprimary color mixing, two configurations of a full-color luminaire made of red, green and blue LEDs are considered. The color parameters for the primary lights are set in Table S2.

Table S2. The color parameters of two configurations of a full-color luminaire

LEDs	Red LEDs			Green LEDs			Blue LEDs		
Para.	*x*	*y*	Flux/lm	*x*	*y*	Flux/lm	*x*	*y*	Flux/lm
Config. *a*	0.6763	0.2837	200	0.1833	0.7303	200	0.1495	**0.0659**	200
Config. *b*	0.6763	0.2837	200	0.1833	0.7303	200	0.1495	**0.1087**	200

3D gamut boundaries of two configurations *a* and *b* in the CIE 1931 *xy*Y color space are plotted in Fig. S3(a) and Fig. S3(b), respectively. As can be seen, the two configurations keep the same luminosities but different chromaticity coordinates of blue LEDs (the other two primaries keep the same). Granted, the configuration *a* will have a relative larger realizable color space, considering a larger 2D gamut of blue primary. In contrast, the codable ratios for configuration *a* and *b* are calculated as 0.494 and 0.513, respectively, in accordance with the equation ⑪ in the main text. Therefore, configuration *b* has a larger realizable color volume and thus a greater tunable capability.

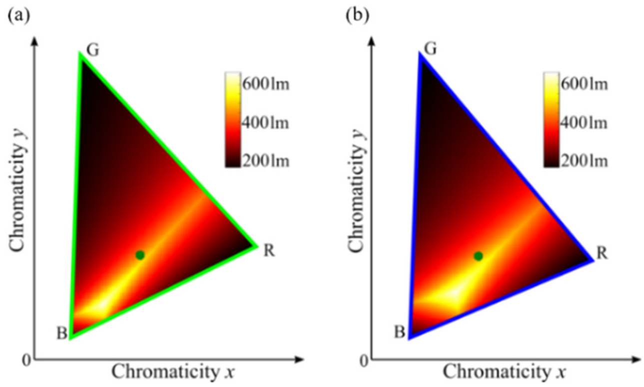

Fig. S3. 3D theoretical and accessible spaces of two configurations for triprimary mixing luminaire. The triangles are the 2D gamut boundaries of CIE *xy*Y color space projected in x-y plane. The color contrasts present the upper boundaries of the chromaticity dependent luminosities of the mixed light. The green points in (a) and (b) are the same reference chromaticity coordinates.

6. Biprimary color mixing

An office luminaire is made of two types of LEDs, namely “cold” and “warm” white LEDs. Each type (color) of white LEDs consists of 8 same LED chips (XP-E2 from CREE Xlamp), which are connected in series. LEDs with the different colors are connected in parallel, which are driven by two independent constant current pulses (driving IC: UCS512C4 from Shenzhen UCS Co., Ltd, current amplitude: 850 mA). The two colors of LEDs are interdigitated and covered with a light mixture to minimize the effect of non-uniformity of exitance. The luminaire is then placed in the center of an integrating sphere (1.5m_R98_V3 from Everfine Co., Ltd). The emitting spectra are measured via a spectrometer (HAAS-2000 from Everfine Co., Ltd) at the exit of the integrating sphere. The color information of each type of white LEDs when the duty cycle reaches 100% is obtained in Table S3.

Table S3. The color parameters of two primary LEDs of an office luminaire

Parameters	x	y	Flux/lm	CCT/K
Cold white LEDs	0.3176	0.3277	1127	6248
Warm white LEDs	0.4731	0.4079	999.3	2505

To realize the expecting coordinates in the 2D CIE 1931 xyY color space, it is worthy to firstly verify whether the points are located in accessible space. The duty cycles of digital pulses for driving each primary light are calculated in accordance with equation ⑤ in the main text. The expecting CCT, chromaticity and luminosity of additive mixed light and calculated duty cycles for each primary light in Fig. 2(b) of the main text are listed in Table S4.

Table S4. The inputs and outputs of biprimary color mixing

No.	x	y	CCT/K	Flux/lm	D_c/%	D_w/%
1	0.3222	0.3301	6000	800.02	68.38	2.94
2	0.3261	0.3321	5800	999.95	82.75	6.74
3	0.3381	0.3383	5250	1146.45	85.57	18.22
4	0.3582	0.3486	4500	1199.9	73.98	36.64
5	0.3854	0.3627	3750	1146.46	51.85	56.25
6	0.4138	0.3773	3200	1000.03	29.37	66.95
7	0.4274	0.3843	3000	799.99	17.80	59.98
8	0.4138	0.3773	3200	600	17.62	40.17
9	0.3854	0.3627	3750	453.7	20.52	22.26
10	0.3581	0.3486	4500	399.93	24.66	12.21
11	0.3381	0.3383	5250	453.65	33.86	7.21
12	0.3261	0.3321	5800	599.93	49.65	4.04
13	0.3222	0.3301	6000	800.02	68.38	2.94

In the Table S4, D_c and D_w are the duty cycles of driving pulses for the "cold" and "warm" white LEDs, respectively. The blue cells indicate the expecting quantities in the CIE 1931 xyY color space, and the green cells indicate the decoded duty cycles of digital pulses.

7. Convert CCTs to chromaticity coordinates

As shown in Fig. S4, the chromaticity coordinates of the "cold" and "warm" white LEDs are C (x_c, y_c) and W (x_w, y_w), respectively. The corresponding CCTs are T_c (blue dashed line) and T_w (red dashed line), respectively. In accordance with the principle of additive color mixing, the chromaticity coordinates of mixed light M (x_m, y_m) should locate on the line segment CW (green line) [1]. Similarly, the CCT of mixed light T_m should be in the range of [T_c, T_w] [1]. For a given T_m, the iso-CCT line has a unique intersection M (x_m, y_m) with the line segment CW. The intersection is the chromaticity coordinates of mixed light that corresponds to the CCT of T_m. Therefore, there is a unique chromaticity coordinate corresponding to an expected CCT for biprimary color mixing.

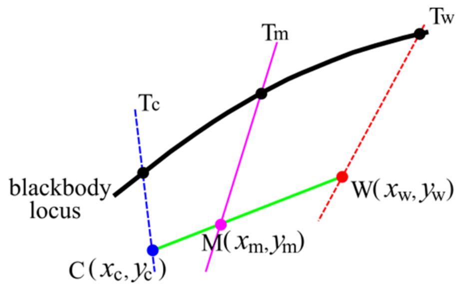

Fig. S4. Seek chromaticity coordinates for a given CCT in biprimary color mixing. The black line represents the blackbody locus in the CIE 1931 chromaticity diagram. The blue and red dashed lines indicate the iso-CCT lines corresponding to the "cold" and "warm" white LEDs, respectively. The magenta line indicates the iso-CCT line of additive mixed light. C (x_c, y_c), W (x_w, y_w) and M (x_m, y_m) represent the chromaticity coordinates of "cold" white LEDs, "warm" white LEDs and additive mixed light.

8. Triprimary color mixing

A full-color luminaire is made of three primary LEDs (LUXEON 2835 from Lumileds): red, green and blue LEDs. Each primary LEDs consist of 9 same LED chips, which are connected in series. LEDs with the different colors are connected in parallel, which are driven by three independent constant current pulses (driving IC: UCS512C4 from Shenzhen UCS Co., Ltd, current amplitude: 850 mA). The measurements are conducted following the same process of biprimary color mixing. The color information of each primary LEDs when the duty cycle reaches 100% is obtained in Table S5.

Table S5. The color parameters of three primary LEDs of a full-color luminaire

Parameters	x	y	Flux/lm
Red LEDs	0.6925	0.3068	168
Green LEDs	0.1675	0.7034	345.69
Blue LEDs	0.1169	0.1087	153.59

To realize the expecting coordinates in the 3D CIE 1931 xyY color space, it is worthy to firstly verify whether the points are located in accessible space. The duty cycles of digital pulses for driving each primary light are calculated in accordance with equation ④ in the main text. The expecting chromaticity and luminosity of additive mixed light and calculated duty cycles for each primary light in Fig. 2(e) of the main text are listed in Table S6.

Table S6. The inputs and outputs of triprimary color mixing

No.	x	y	Flux/lm	D_r/%	D_g/%	D_b/%
1	0.5078	0.3774	280	89.26	35.08	5.72
2	0.3278	0.5574	280	28.36	66.59	1.4
3	0.1477	0.3754	280	1.95	67.48	28.29
4	0.3278	0.1973	280	94.3	8.03	61.07
5	0.4678	0.4402	280	67.09	47.25	2.58
6	0.3906	0.5173	280	42.27	59.98	1.06
7	0.265	0.5174	280	20.04	68.24	6.8
8	0.1878	0.4402	280	8.88	68.85	17.63
9	0.1878	0.3145	280	15.54	56.92	37.19
10	0.2649	0.2373	280	52.86	32.31	51.77

11	0.3888	0.2369	280	100	14.32	39.24
12	0.4678	0.3145	280	97.01	26.68	16.13
13	0.4152	0.4648	280	52.77	53.81	3.47
14	0.4152	0.2899	280	89.29	26.74	24.44
15	0.2404	0.4647	280	18.35	66.59	12.37
16	0.2404	0.2899	280	34.11	47.22	38.7

In Table S6, D_r, D_g and D_b represent the duty cycles of driving pulses for the red, green and blue LEDs, respectively. The blue cells indicate the expecting quantities in the CIE 1931 *xy*Y color space, and the green cells indicate the decoded duty cycles of digital pulses.

9. Reconstruct digital frames

An image displayed in a digital device is usually coded in sRGB color space. Thus, the first step is to transform a digital image from sRGB to the CIE 1931 *XYZ* tristimulus space [2]. Notably, the gamma corrected RGB map should be linearized first. Next, the CIE 1931 *XYZ* tristimulus space is transformed to the CIE 1931 *xy*Y color space [2]. By using the decoding algorithm, the digital signals of three primaries for each pixel can be obtained. The full-color luminaire made of three primary LEDs is put inside an integrating sphere (1.5m from Everuping Optics Co., Ltd) and then driven by three independent current pulses as calculated. The emission spectra of the mixed light are measured (EU2000 CCD spectrometer from Everuping Optics Co., Ltd) at the exit of the integrating sphere. The chromaticity and luminosity in the CIE 1931 *xy*Y color space are extracted from the spectrum, corresponding to the color of a single pixel of the frame. A CIE 1931 *xy*Y color map can be obtained by repeating the same procedure for a loop that corresponds to the total number of pixels of the reconstructing image. In order to display the color map in a digital device and compare with the original image, we transform the color map from the CIE 1931 *xy*Y color space back to sRGB color space [2].

10. Laser beam scanning projector

The color parameters of cancer-horoscopic-like chromatic pattern shown in Fig. 3(b) of main text are listed as follows:

Table S7. Chromaticity coordinates of seven colors

Color	*x*	*y*	Flux/lm
1	0.5354	0.3190	320
2	0.1591	0.5967	320
3	0.2058	0.1451	320
4	0.3001	0.3536	320
5	0.2755	0.3960	320
6	0.3706	0.2321	320
7	0.3943	0.3536	320

In order to realize the chromaticity and luminosity listed above, a RGB laser combining three separate monochromatic beams is used. The color parameters of individual laser are measured as follows:

Table S8. The color parameters of RGB laser

Parameters	Central λ/ nm	*x*	*y*	Flux/lm
Red laser	637	0.65304	0.30174	321.0926
Green laser	515	0.08862	0.71828	1581.053

Blue laser	450	0.15868	0.04084	348.058

The intensity of individual laser is modulated by TTL pulses, which are communicated with the computer via a serial port. The experimental setups are shown in Fig. S5:

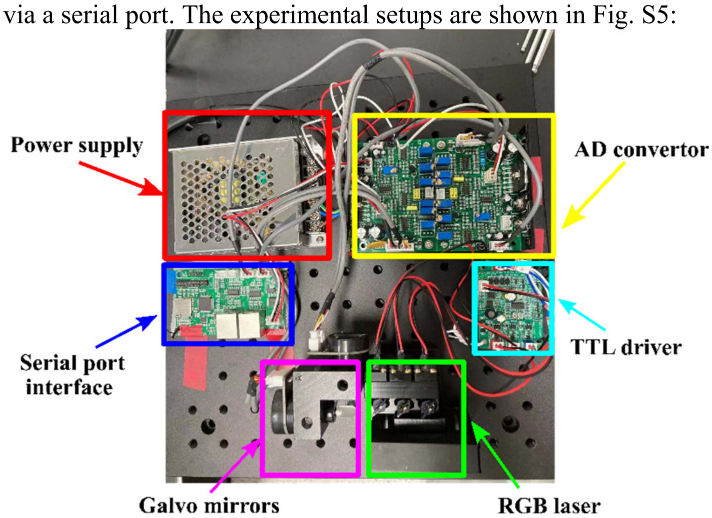

Fig. S5. Experimental setups of home-built laser beam scanning projector

The mixed laser beam with precise chromaticity and luminosity is reflected by two orthogonal galvo scanning mirrors, and finally projected to a vertical screen. The scanning images shown in Fig. 3(b) of the main text are measured in a dark room by using a digital camera (HUAWEI-nova 4) in a distance of 1 m away from the screen.

11. Nanostructure fabrication

Cleaned by acetone and isopropanol (IPA) in an ultrasonic bath, the Indium-tin-oxide (ITO) glass (ITO thickness 180 nm) was spin-coated (spinning speed: 5000 r/s, acceleration: 2500 r/s^2, duration: 60 s) with the positive resist (PMMA 950K, AR-P672.045, Allresist, Germany). The thickness of the resist is expected to be 198 ± 5 nm, according to the empirical curve of the spinning speed versus the film thickness. After soft baking (hot plate at 175 °C for 3 minutes), the resist was exposed with the pre-designed patterns by an electron beam. (ZEISS Crossbeam 550L) The written patterns were developed via developer (MIBK: IPA = 1:3) for 60 s, followed by the stopping step dipping the patterns in IPA for 60 s. In the development, the exposed areas of the resist were removed. Subsequently, high vacuum evaporation process was then used to coat the sample with a Cr layer of 3 nm (for adhesion purpose) and the target metal Au film of 50 nm. The sample was then immersed in acetone for more than 24 hours to dissolve the unexposed resist. The sample was finally obtained following a standard lift-off procedure.

12. Theoretical chromaticity coordinates calculations

In order to calculate the theoretical chromaticity of transmission light passing though plasmonic metasurfaces with different dimensions, three steps are followed:

(1) Laser spectra calculation

The laser spectra could be regarded as Gaussian pulses with central wavelength at 450 nm, 515 nm and 637 nm respectively. The pulse width is extracted from experimental spectra: 6 nm. The amplitudes of pulses are obtained from the emission spectra when D_i=100% (i=1,2,3). During the sensing process, the laser changes the amplitudes of three primary lasers in accordance with the calculated duty cycles, since the amplitude is proportional to the duty cycles.

(2) Transmission spectra using white light

The transmission spectra of plasmonic metasurfaces are calculated using Finite-Difference Time-Domain (FDTD) method. The established model is shown in Fig. S6:

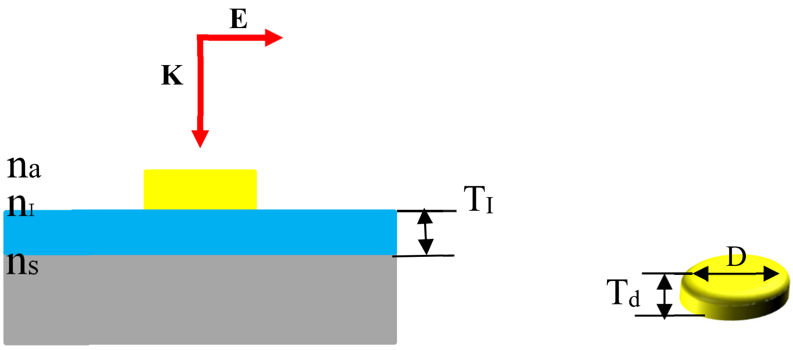

Fig. S6. FDTD model

Au nanodisc is located on a glass substrate (n_s= 1.45) with a 180 nm thickness of ITO layer (n_I= 1.88) in between. A plane wave with a normal incidence and a linear polarization along the *x* axis comes from the air (n_a= 1). The spectrum covers the whole visible regime (380 nm-780 nm). The nanodisc has the same height: T_d= 48 nm and the diameter varies from 60 nm to 140 nm in a step of 10 nm. The period of nanodisc array is set the same as the observation of SEM images: 200 nm. The calculated transmission spectra of plasmonic metasurfaces are shown in Fig. S7.

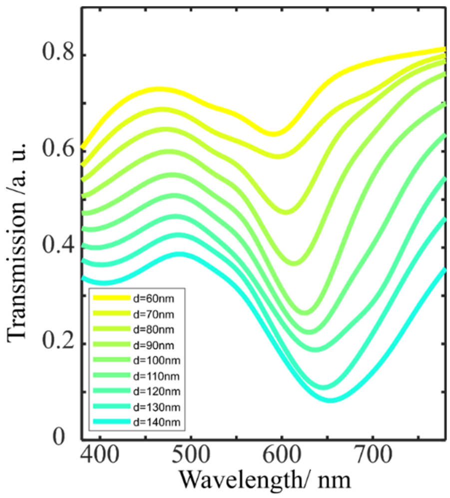

Fig. S7. Theoretical transmission spectra of plasmonic metasurfaces

(3) Chromaticity calculation

The transmission spectra using RGB laser as the light source can be simply obtained by multiplying the laser spectra calculated in step (1) and transmission spectra of metasurfaces calculated in step (2). The chromaticity coordinates can thus be calculated from the transmission spectra in accordance with the CIE 1931 standard [1].

References

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