Supplemental Document

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Dual-dispersion-regime dual-comb mode-locked laser: supplement

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1. NOISE CHARACTERISTICS

The different pulse shaping mechanisms of the dual-dispersion dual-comb laser presented in the primary document may influence the relative noise characteristics compared to the original all-anomalous-dispersion dual-comb laser design of ref. [1]. In particular, it is unclear how the two setups compare in terms of common phase noise suppression relevant for mode-resolved dual-comb spectroscopy. Because the relative timing and carrier-envelope offset (CEO) noise can be inferred from dual-comb interferograms, we processed 200-ms-long acquisitions using a computational phase retrieval and correction algorithm [2] to compare the two setups.

First we compare the timing stability, where the nomenclature of ref. [3] by Camenzind et al. is adopted. The relative root-mean-square (RMS) timing jitter τ^{RMS} is calculated from the n-th interferogram arriving at t_n, and compared to the expected time-of-arrival (TOA) defined by multiples of the expected period $1/\left\langle \Delta f_{rep} \right\rangle$:

$$\tau^{\mathrm{RMS}}[n] = t_n - \frac{n}{\left\langle \Delta f_{rep} \right\rangle}. \tag{S1}$$

This quantity characterizes how much the delay between optical pulses from the two combs fluctuates compared to the jitter-free case. To retrieve the TOAs needed for the characterization, we employ a constant fraction discriminator [2], which provides a numerical trigger based on the digital difference frequency signal (DDFG) [4]. However, other techniques should serve this application equally well like detecting peaks of the dual-comb signal envelope [3] or finding the maximum of the cross-correlation/cross-ambiguity function [5,6]. Note that to relate the microwave and optical domains, we employ a comb factor $\Delta f_{\mathrm{rep}}/f_{\mathrm{rep}}$ in our calculations, as in ref. [3].

Another related quantity under study is the period timing jitter, which measures how much the duration of interferograms (difference of consecutive TOAs) fluctuates relative to the expected period. It is defined as

$$\tau^{\mathrm{period}}[n] = (t_n - t_{n-1}) - \frac{1}{\left\langle \Delta f_{\mathrm{rep}} \right\rangle}. \tag{S2}$$

Table S1 summarizes the numerical results obtained for the two lasers.

Table S1. Comparison between the all-anomalous and dual-dispersion mode of operation of the dual-comb mode-locked laser utilizing a Yb:CNGS gain medium.

	Single-dispersion (all-anomalous) regime [1]	Dual-dispersion-regime
Relative RMS timing jitter between the two pulse trains	20.7 ps	2.5 ps
Period timing jitter	331.3 fs	108.8 fs

Unexpectedly, we find that both the relative RMS timing jitter, and period timing jitter greatly improve when we employ the dual-dispersion setup. Consequently, the latter is better suited for free-running asynchronous optical sampling or dual-comb spectroscopy experiments. The exact origin of the noise performance improvement needs further research but we postulate that it may relate to (i) reduced spectral overlap between the combs; (ii) lower peak power of the multi-ps pulse that forms in the normal dispersion regime. When the two pulses temporally overlap inside the gain crystal, they compete for the gain. This often leads to severe pulse amplitude modulation, which manifests itself in polarization-multiplexed fiber lasers as slowly-varying post-centerburst oscillations in the dual-comb interferogram [7]. Here, the spectral overlap between the *o*- and *e*-beams around their respective peak wavelengths is significantly smaller when compared to ref. [1], which might have contributed to decreasing their gain-competition-based cross-coupling. Additionally, by lengthening the duration of one of the pulses to the multi-picosecond range, we expect to have greatly suppressed intensity-dependent inter-pulse nonlinear interaction.. Note that it is minor sub-Hz Δf_{rep} drifts that are mostly responsible for the higher jitter reported here, which correspond to a relative f_{rep} stability as low as 10^{-8}. We also want to underline that the numbers quoted for the two free-running dual-comb laser setups are not a limitation *per se,* and can be greatly improved with a more robust mechanical cavity design [3].

Equally important for performance evaluation is the phase of the relative carrier-envelope-offset (CEO) frequency (Δf_{CEO}), or simply the interferogram carrier-envelope phase $\Delta\varphi_0$. For a stable dual-comb laser with $\Delta f_{\text{CEO}} \neq 0$, $\Delta\varphi_0$ is expected to evolve linearly. Unfortunately, a free-running non-CEO-stabilized dual-comb laser exhibits highly nonlinear phase excursions instead. In Fig. S1, we characterize the power spectral density (PSD) of computationally-retrieved $\Delta\varphi_0$ compensated for the linear trend. The top panel illustrates the frequency spectrum (power spectral density) of phase excursions, while the bottom shows the frequency-integrated phase noise retrieved from the PSD ($\int S_{\Delta fceo}$). Note that $\Delta\varphi_0$ is sampled only around interferogram centerbursts with a sampling frequency of Δf_{rep}, which limits the analysis frequency range to $\Delta f_{\text{rep}}/2$.

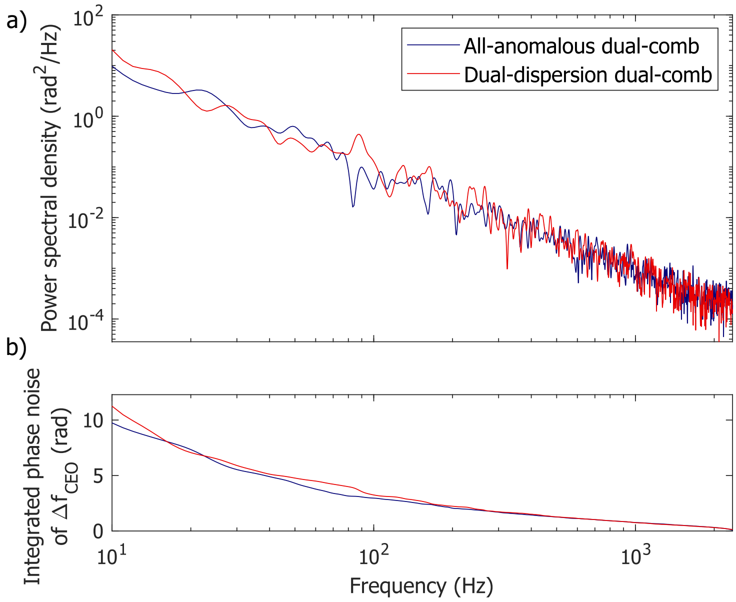

Fig. S1. Relative offset frequency noise characterization retrieved from dual-comb interferograms for the anomalous-only dispersion regime laser [1], and the dual-dispersion regime laser discussed in the main manuscript. (a) Power spectral density (PSD) of relative CEO phase fluctuations. (b) Integrated phase noise retrieved from the PSD.

Unlike in the timing jitter case, we do not see significant differences between the two laser configurations. Both display nearly the same relative carrier-envelope phase noise characteristics with a dominant $1/f^2$ slope (white frequency noise rolling off −20 dB/decade) and only minor local differences occurring at acoustic/mechanical noise frequencies. This further confirms the importance of stable mechanical cavity design for low-timing-jitter and low-phase-noise operation, which may be potentially degraded by environmental noise effects.

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