Supplemental Document

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Ultra-wideband linear-to-circular polarizer realized by bi-layer metasurfaces: supplement

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Ultra-wideband Linear-to-Circular Polarizer Realized By Bi-layer Metasurfaces: supplemental document

To analyze the near-field coupling between two metasurfaces layers in detail, we decompose the proposed bi-layer metasurface [see Fig. 1(b)] to a four-layer system, as presented in Fig. 13(a). In the four-layer system, layer 1 and layer 2 (layer 3 and layer 4) are respectively spaced by an air gap with thickness of t_0 (t_0=0). When this four-layer system is normally illuminated by a x-polarized plane wave, each metasurface layer will generate induced currents, as presented in Fig. S1(b). According to ref. [1], the induced current density vector ($\vec{J}_p(\rho,\omega)$) of layer p (p=1,2,3,4) is denoted as

$$\begin{cases} \vec{J}_1(\rho,\omega) = c_1(\omega)\vec{J}_1(\rho) \\ \vec{J}_2(\rho,\omega) = c_2(\omega)\sum_l \vec{J}_2(\rho-\rho_l) \\ \vec{J}_3(\rho,\omega) = c_2(\omega)\sum_l \vec{J}_2(\rho-\rho_l) \\ \vec{J}_4(\rho,\omega) = c_1(\omega)\vec{J}_1(\rho) \end{cases} \tag{S1}$$

where $\vec{J}_1(\rho)$ is the induced current density vector on the metal strip along the x-direction, and $\sum_l \vec{J}_2(\rho-\rho_l)$ (l = 1, 2, 3, 4) is the induced current density vector on Jerusalem-cross-like resonator [see layer 2 in Fig. S1(b)]. It is noted that $c_{s(s=1,2)}(\omega)$ is frequency-dependent complex amplitude coefficient, $\vec{J}_{s(s=1,2)}(\rho)$ is the spatial profile of current, and $\rho_{l(l=1,2,3,4)}$ is the position of the lth scatterer. The currents $\vec{J}_1(\rho)$ and $\vec{J}_2(\rho\text{-}\rho_l)$ are independent of ω and they can be solved by using the eigenmode solver in commercial software CST Microwave Studio.

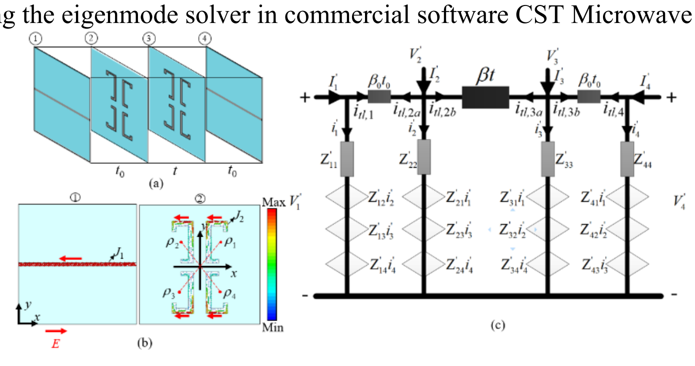

Fig. S1. (a) The decomposed four-layer metasurface. (b) Surface current distributions of layer 1 and layer 2. (c) The equivalent circuit model of the four-layer system.

The self-impedance in Fig. S1(c) can be written as

$$\left. \begin{array}{l} Z'_{ii(i=1,2)} = \sum\limits_{qmn(m,n)\neq(0,0)} \dfrac{M_{i,q}^2\left[\left(Y_q^1+Y_q^2\right)-\left(Y_q^1\text{-}Y_q^2\right)e^{-2j\beta_{mn}t}\right]}{\left(Y_q^1+Y_q^2\right)^2-\left(Y_q^1\text{-}Y_q^2\right)^2 e^{-2j\beta_{mn}t}} \\ Z'_{11}=Z'_{44}, Z'_{22}=Z'_{33} \end{array} \right\}, \tag{S2}$$

and the mutual impedance can be written as

$$\left.\begin{array}{l}
Z_{12}^{'}=Z_{21}^{'}=Z_{34}^{'}=Z_{43}^{'}\\
Z_{12}^{'}=\sum\limits_{qmn(m,n)\neq(0,0)}\dfrac{M_{1,q}M_{2,q}^{*}2Y_q^2}{\left(Y_q^1+Y_q^2\right)^2}\\
Z_{14}^{'}=Z_{41}^{'},\ \ Z_{23}^{'}=Z_{32}^{'}\\
Z_{14}^{'}=\sum\limits_{qmn(m,n)\neq(0,0)}\dfrac{M_{1,q}M_{1,q}^{*}\left(2Y_q^2e^{-j\beta_{mn}t}\right)}{\left(Y_q^1+Y_q^2\right)^2-\left(Y_q^1-Y_q^2\right)^2e^{-2j\beta_{mn}t}}\\
Z_{23}^{'}=\sum\limits_{qmn(m,n)\neq(0,0)}\dfrac{M_{2,q}M_{2,q}^{*}\left(2Y_q^2e^{-j\beta_{mn}t}\right)}{\left(Y_q^1+Y_q^2\right)^2-\left(Y_q^1-Y_q^2\right)^2e^{-2j\beta_{mn}t}}\\
M_{1,q}=\left(\tilde{J}_1(\mathrm{k}_{t,q})\cdot\vec{e}_q\right)/\left(\tilde{J}_1(\mathrm{k}_0)\cdot\vec{e}_q\right),\ \ M_{2,q}=\left(\tilde{J}_2(\mathrm{k}_{t,q})\cdot\vec{e}_q\right)/\left(\tilde{J}_2(\mathrm{k}_0)\cdot\vec{e}_q\right)
\end{array}\right\}, \tag{S3}$$

where β_{mn} is the propagation constants of nmth harmonics(m,n=0,), Y_q^1 and Y_q^2 is the admittance of the Floquet harmonic in free space and in dielectric layer, respectively. For TM harmonics, $\vec{e}_q=\left(k_{xn}\vec{e}_x+k_{ym}\vec{e}_y\right)\Big/\sqrt{k_{xn}^2+k_{ym}^2}$, and for TE harmonics, $\vec{e}_q=\left(k_{ym}\vec{e}_x-k_{xn}\vec{e}_y\right)\Big/\sqrt{k_{xn}^2+k_{ym}^2}$. $\tilde{J}_1\left(\tilde{J}_2\right)$ is the Fourier transform of the current on layer 1(layer 2),

$$\tilde{J}_i(k_{t,q})=\int_{u,c.}\vec{J}_i(\rho)e^{jk_{t,q}\rho}d\rho \tag{S4}$$

Similar to Eq. (2) in Main Text, equivalent voltages $V_n^{'}$ and $i_n^{'}$ (n = 1, 2, 3, 4) are also denoted by the near-field impedance matrix $\left[Z^{'}\right]$,

$$\begin{bmatrix}i_1^{'}\\i_2^{'}\\i_3^{'}\\i_4^{'}\end{bmatrix}=\begin{bmatrix}Z_{11}^{'}&Z_{12}^{'}&Z_{13}^{'}&Z_{14}^{'}\\Z_{21}^{'}&Z_{22}^{'}&Z_{23}^{'}&Z_{24}^{'}\\Z_{31}^{'}&Z_{32}^{'}&Z_{33}^{'}&Z_{34}^{'}\\Z_{41}^{'}&Z_{42}^{'}&Z_{43}^{'}&Z_{44}^{'}\end{bmatrix}^{-1}\begin{bmatrix}V_1^{'}\\V_2^{'}\\V_3^{'}\\V_4^{'}\end{bmatrix}=\begin{bmatrix}Y_{11}^{'}&Y_{12}^{'}&Y_{13}^{'}&Y_{14}^{'}\\Y_{21}^{'}&Y_{22}^{'}&Y_{23}^{'}&Y_{24}^{'}\\Y_{31}^{'}&Y_{32}^{'}&Y_{33}^{'}&Y_{34}^{'}\\Y_{41}^{'}&Y_{42}^{'}&Y_{43}^{'}&Y_{44}^{'}\end{bmatrix}\begin{bmatrix}V_1^{'}\\V_2^{'}\\V_3^{'}\\V_4^{'}\end{bmatrix}. \tag{S5}$$

Owing to t_0=0, layer 1 and layer 2 are overlapped. Hence the voltage $V_1^{'}$ is approximately equal to $V_2^{'}$. Similarly, $V_3^{'}$=$V_4^{'}$. In this case (S5) can be written as

$$\begin{bmatrix}i_1^{'}\\i_2^{'}\\i_3^{'}\\i_4^{'}\end{bmatrix}=\begin{bmatrix}Y_{11}^{'}&Y_{12}^{'}&Y_{13}^{'}&Y_{14}^{'}\\Y_{21}^{'}&Y_{22}^{'}&Y_{23}^{'}&Y_{24}^{'}\\Y_{31}^{'}&Y_{32}^{'}&Y_{33}^{'}&Y_{34}^{'}\\Y_{41}^{'}&Y_{42}^{'}&Y_{43}^{'}&Y_{44}^{'}\end{bmatrix}\begin{bmatrix}V_1^{'}\\V_1^{'}\\V_4^{'}\\V_4^{'}\end{bmatrix}, \tag{S6}$$

Through a simple calculation process, Eq. (S6) can be reduced into a 2×2 matrix.

$$\begin{bmatrix}i_a\\i_b\end{bmatrix}=\begin{bmatrix}i_1^{'}+i_2^{'}\\i_3^{'}+i_4^{'}\end{bmatrix}=\begin{bmatrix}Y_{11}^{eq}&Y_{12}^{eq}\\Y_{21}^{eq}&Y_{22}^{eq}\end{bmatrix}\begin{bmatrix}V_1^{'}\\V_4^{'}\end{bmatrix}=\begin{bmatrix}Z_{11}^{eq}&Z_{12}^{eq}\\Z_{21}^{eq}&Z_{22}^{eq}\end{bmatrix}^{-1}\begin{bmatrix}V_1^{'}\\V_4^{'}\end{bmatrix}, \tag{S7}$$

where $Y_{11}^{eq}=Y_{11}^{'}+Y_{12}^{'}+Y_{21}^{'}+Y_{22}^{'}$, $Y_{12}^{eq}=Y_{13}^{'}+Y_{14}^{'}+Y_{23}^{'}+Y_{24}^{'}$, $Y_{21}^{eq}=Y_{31}^{'}+Y_{32}^{'}+Y_{41}^{'}+Y_{42}^{'}$, and $Y_{22}^{eq}=Y_{33}^{'}+Y_{34}^{'}+Y_{43}^{'}+Y_{44}^{'}$. Actually, Eq. (S6) is identical with Eq. (2). The currents i_a and i_b are respectively equal to i_1 and i_2 in Eq. (2). Correspondingly, the voltages $V_1^{'}$ and $V_4^{'}$ are respectively equal to V_1 and V_2, and $\begin{bmatrix}Y_{11}^{eq}&Y_{12}^{eq}\\Y_{21}^{eq}&Y_{22}^{eq}\end{bmatrix}$ is equal to $\begin{bmatrix}Y_{11}&Y_{12}\\Y_{21}&Y_{22}\end{bmatrix}$ in Eq. (2). Up to now the [Z] matrix presented in Eq. (2) can be obtained from the $\left[Y^{'}\right]$ matrix shown in Eq. (S5). Unlike those reported methods, we can obtain the S parameters of multilayer metasurfaces only by calculating the current distributions of each single-layer metasurface, thereby effectively decreasing the requirement of computer.

References

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