Supplemental Document

Optics Letters

Probing coherence Stokes parameters of three-component light with nanoscatterers: supplement

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Supplement DOI: https://doi.org/10.6084/m9.figshare.19619865

Parent Article DOI: https://doi.org/10.1364/OL.457963

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This supplementary document provides detailed derivations of the difference between the distances from the dipoles to the far-zone point as well as Eqs. (16a)–(16i) appearing in the main text.

DERIVATION OF DISTANCE DIFFERENCE

Suppose that the two dipoles are located symmetrically about the origin along the x axis with separation $d = 2a$ (see Fig. 1). In the Cartesian coordinate frame we then have $\mathbf{r}_1 = (-a, 0, 0)$, $\mathbf{r}_2 = (a, 0, 0)$, and $\mathbf{r} = (r\sin\theta\cos\varphi, r\sin\theta\sin\varphi, r\cos\theta)$, where $r = |\mathbf{r}|$. Hence,

$$\begin{aligned} r_1 &= \sqrt{(r\sin\theta\cos\varphi + a)^2 + (r\sin\theta\sin\varphi)^2 + (r\cos\theta)^2} \\ &= r\sqrt{1 + \left(\frac{a}{r}\right)^2 + 2\frac{a}{r}\sin\theta\cos\varphi}. \end{aligned} \tag{S1}$$

Since $r \gg a$, we can neglect the second term and use $(1+x)^{1/2} \approx 1 + x/2$ to obtain

$$r_1 \approx r\big(1 + \frac{a}{r}\sin\theta\cos\varphi\big). \tag{S2}$$

In a similar way we find that

$$\begin{aligned} r_2 &= \sqrt{(r\sin\theta\cos\varphi - a)^2 + (r\sin\theta\sin\varphi)^2 + (r\cos\theta)^2} \\ &\approx r\big(1 - \frac{a}{r}\sin\theta\cos\varphi\big). \end{aligned} \tag{S3}$$

Equations (S2) and (S3) imply at once that

$$k(r_1 - r_2) \approx 2ka\sin\theta\cos\varphi = kd\sin\theta\cos\varphi, \tag{S4}$$

which was employed in the discussion above Eq. (14) of the main text.

DERIVATION OF EQUATIONS (16a)–(16i)

The three observation directions (θ, φ) under consideration are $(\pi/2, 0)$, $(\pi/2, \pi/2)$, and $(0, \pi/2)$, as illustrated in Fig. 2.

Direction $(\pi/2, 0)$*:* From Eqs. (5) and (6) one finds that $E'_\theta(\mathbf{r}_i, \omega) = -E_z(\mathbf{r}_i, \omega)$ and $E'_\varphi(\mathbf{r}_i, \omega) = E_y(\mathbf{r}_i, \omega)$, which are the field components at the dipole location $\mathbf{r}_i$ along the $\hat{\mathbf{u}}_\theta$ and $\hat{\mathbf{u}}_\varphi$ directions, respectively. Thus the elements of the dipole-site 2×2 cross-spectral density matrix constructed from the components transverse to the direction $(\pi/2, 0)$ are related to the Cartesian-system elements as

$$\begin{bmatrix} W'^{(12)}_{\theta\theta} & W'^{(12)}_{\theta\varphi} \\ W'^{(12)}_{\varphi\theta} & W'^{(12)}_{\varphi\varphi} \end{bmatrix} = \begin{bmatrix} W^{(12)}_{zz} & -W^{(12)}_{zy} \\ -W^{(12)}_{yz} & W^{(12)}_{yy} \end{bmatrix}, \tag{S5}$$

where the superscript (12) refers to points $\mathbf{r}_1$ and $\mathbf{r}_2$. Using Eqs. (13a), (13c), and (13d) then yields

$$S'^{(12)}_0(\frac{\pi}{2}, 0) = W^{(12)}_{zz} + W^{(12)}_{yy}, \tag{S6}$$

$$S'^{(12)}_2(\frac{\pi}{2}, 0) = -[W^{(12)}_{yz} + W^{(12)}_{zy}], \tag{S7}$$

$$S'^{(12)}_3(\frac{\pi}{2}, 0) = \mathrm{i}[W^{(12)}_{yz} - W^{(12)}_{zy}]. \tag{S8}$$

On further substituting Eqs. (S7) and (S8) into Eqs. (2g) and (2h), respectively, one finds

$$\Lambda_6^{(12)} = -S_2'^{(12)}(\frac{\pi}{2}, 0), \tag{S9}$$

$$\Lambda_7^{(12)} = S_3'^{(12)}(\frac{\pi}{2}, 0), \tag{S10}$$

which are Eqs. (16g) and (16h) of the main text.

Direction $(\pi/2, \pi/2)$: Now $E'_\theta(\mathbf{r}_i, \omega) = -E_z(\mathbf{r}_i, \omega)$ and $E'_\varphi(\mathbf{r}_i, \omega) = -E_x(\mathbf{r}_i, \omega)$, whereupon

$$\begin{bmatrix} W'^{(12)}_{\theta\theta} & W'^{(12)}_{\theta\varphi} \\ W'^{(12)}_{\varphi\theta} & W'^{(12)}_{\varphi\varphi} \end{bmatrix} = \begin{bmatrix} W^{(12)}_{zz} & W^{(12)}_{zx} \\ W^{(12)}_{xz} & W^{(12)}_{xx} \end{bmatrix}. \tag{S11}$$

The use of Eqs. (13a), (13c), and (13d) subsequently results in

$$S_0'^{(12)}(\frac{\pi}{2}, \frac{\pi}{2}) = W^{(12)}_{zz} + W^{(12)}_{xx}, \tag{S12}$$

$$S_2'^{(12)}(\frac{\pi}{2}, \frac{\pi}{2}) = W^{(12)}_{xz} + W^{(12)}_{zx}, \tag{S13}$$

$$S_3'^{(12)}(\frac{\pi}{2}, \frac{\pi}{2}) = -\mathrm{i}[W^{(12)}_{xz} - W^{(12)}_{zx}]. \tag{S14}$$

By then inserting Eqs. (S13) and (S14) into Eqs. (2e) and (2f), respectively, one obtains

$$\Lambda_4^{(12)} = S_2'^{(12)}(\frac{\pi}{2}, \frac{\pi}{2}), \tag{S15}$$

$$\Lambda_5^{(12)} = -S_3'^{(12)}(\frac{\pi}{2}, \frac{\pi}{2}), \tag{S16}$$

which are Eqs. (16e) and (16f) of the body text.

Direction $(0, \pi/2)$: In this case $E'_\theta(\mathbf{r}_i, \omega) = E_y(\mathbf{r}_i, \omega)$ and $E'_\varphi(\mathbf{r}_i, \omega) = -E_x(\mathbf{r}_i, \omega)$. Therefore,

$$\begin{bmatrix} W'^{(12)}_{\theta\theta} & W'^{(12)}_{\theta\varphi} \\ W'^{(12)}_{\varphi\theta} & W'^{(12)}_{\varphi\varphi} \end{bmatrix} = \begin{bmatrix} W^{(12)}_{yy} & -W^{(12)}_{yx} \\ -W^{(12)}_{xy} & W^{(12)}_{xx} \end{bmatrix}, \tag{S17}$$

and following a similar procedure as above leads to

$$S_0'^{(12)}(0, \frac{\pi}{2}) = W^{(12)}_{xx} + W^{(12)}_{yy}, \tag{S18}$$

$$S_2'^{(12)}(0, \frac{\pi}{2}) = -[W^{(12)}_{xy} + W^{(12)}_{yx}], \tag{S19}$$

$$S_3'^{(12)}(0, \frac{\pi}{2}) = \mathrm{i}[W^{(12)}_{xy} - W^{(12)}_{yx}]. \tag{S20}$$

Comparing Eqs. (S19) and (S20) with Eqs. (2b) and (2c) finally reveals that

$$\Lambda_1^{(12)} = -S_2'^{(12)}(0, \frac{\pi}{2}), \tag{S21}$$

$$\Lambda_2^{(12)} = S_3'^{(12)}(0, \frac{\pi}{2}), \tag{S22}$$

which are Eqs. (16b) and (16c) of the paper.

Remaining relations: From Eqs. (2a), (2d), (2i) and (S6), (S12), (S18) one straightforwardly obtains

$$\Lambda_0^{(12)} = \frac{1}{2}\Big[S_0'^{(12)}(\frac{\pi}{2}, 0) + S_0'^{(12)}(\frac{\pi}{2}, \frac{\pi}{2}) + S_0'^{(12)}(0, \frac{\pi}{2})\Big], \tag{S23}$$

$$\Lambda_3^{(12)} = S_0'^{(12)}(\frac{\pi}{2}, \frac{\pi}{2}) - S_0'^{(12)}(\frac{\pi}{2}, 0), \tag{S24}$$

$$\Lambda_8^{(12)} = \frac{1}{\sqrt{3}}\Big[2S_0'^{(12)}(0, \frac{\pi}{2}) - S_0'^{(12)}(\frac{\pi}{2}, \frac{\pi}{2}) - S_0'^{(12)}(\frac{\pi}{2}, 0)\Big], \tag{S25}$$

which are Eqs. (16a), (16d), and (16i).