Supplemental Document

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Measurement of natural frequencies and mode shapes of transparent insect wings using common-path ESPI: supplementary material

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This document provides supplementary information to "Measurement of natural frequencies and mode shapes of transparent insect wings using common-path ESPI". Supplement A is the derivation of modulation function $J_0(\Omega)$ of time-averaged ESPI. Supplement B is the derivation of amplitude modulation function $J_0[(1+\delta)\Omega]-J_0(\Omega)$. And the errors between original and approximate values of amplitude modulation function with different modulation coefficients are shown in fig. S1, i.e., $\{J_0[(1+\delta)\Omega]-J_0(\Omega)\}+2J_1\left[\left(1+\frac{\delta}{2}\right)\Omega\right]\sin\frac{(\delta\Omega)}{2}$, $0.1\le\delta\le1$.

Supplement A：Derivation of time average interferogram of steady state vibration

Parameters: I_O is object light, I_R is reference light, φ is initial phase, Ω is phase amplitude of the vibrating object, ω is angular frequency of vibrating object, τ is camera exposure time ($\approx\frac{2N\pi}{\omega}$), N is the multiple of the exposure time to the vibration period of the object.

The interference light intensity on the camera target surface at time t is：

$$I_O+I_R+2\sqrt{I_OI_R}\cos\left[\varphi+\Omega\sin(\omega t)\right] \tag{S1}$$

The average light intensity of the image taken by the camera is：

$$I=\frac{1}{\tau}\int_0^\tau I_O+I_R+2\sqrt{I_OI_R}\cos\left[\varphi+\Omega\sin(\omega t)\right]dt \tag{S2}$$

$$=(I_O+I_R)+2\sqrt{I_OI_R}\int_0^\tau\cos\left[\varphi+\Omega\sin(\omega t)\right]dt \tag{S3}$$

$$=(I_O+I_R)+2\sqrt{I_OI_R}\cos\varphi\int_0^\tau\cos\left[\Omega\sin(\omega t)\right]dt \tag{S4}$$

Expand $\cos\left[\Omega\sin(\omega t)\right]$ into Taylor series to get：

$$\int_0^\tau\cos\left[\Omega\sin(\omega t)\right]dt \tag{S5}$$

$$=\int_0^\tau\left\{\sum_{n=0}^\infty(-1)^n\frac{\left[\Omega\sin(\omega t)\right]^{2n}}{(2n)!}\right\}dt \tag{S6}$$

$$=\sum_{n=0}^\infty(-1)^n\frac{\Omega^{2n}}{(2n)!}\int_0^\tau\left[\sin(\omega t)\right]^{2n}dt \tag{S7}$$

Substitute integral variable, make $d\mu=\omega dt$:

$$=\sum_{n=0}^\infty(-1)^n\frac{\Omega^{2n}}{(2n)!}\int_0^{2N\pi}\frac{\left[\sin(\mu)\right]^{2n}}{\omega}d\mu \tag{S8}$$

$$=\sum_{n=0}^\infty(-1)^n\frac{\Omega^{2n}}{(2n)!}\frac{(2n-1)!!}{(2n)!!}\frac{2N\pi}{\omega} \tag{S9}$$

$$=\tau\sum_{n=0}^\infty(-1)^n\frac{\Omega^{2n}}{[(2n)!!]^2} \tag{S10}$$

$$=\tau\sum_{n=0}^\infty\frac{(-1)^n}{n!n!}\left(\frac{\Omega}{2}\right)^{2n} \tag{S11}$$

$$= \tau J_0(m) \tag{S12}$$

Thus:

$$I = \frac{1}{\tau}\int_0^{\tau} I_O + I_R + 2\sqrt{I_O I_R}\cos\left[\varphi + \Omega\sin(\omega t)\right] dt$$

$$= I_O + I_R + 2\sqrt{I_O I_R}\cos\varphi J_0(\Omega)$$

Supplement B：Derivation of amplitude modulation formula

Parameters: δ is the amplitude modulation coefficient($|\delta| < 1$), $J_0(\Omega)$is zero-order Bessel function equal to $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!n!}\left(\frac{\Omega}{2}\right)^{2n}$, $J_1(\Omega)$is first-order Bessel function equal to $\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(n+1)!}\left(\frac{\Omega}{2}\right)^{2n+1}$.

The idea of derivation is to construct the same variables, making it easy to merge similar items. The original amplitude modulation formula is:

$$J_0[(1+\delta)\Omega] - J_0(\Omega) \tag{S13}$$

First, let $\begin{cases} A = \left(1 + \frac{\delta}{2}\right)\Omega \\ B = \frac{\delta}{2}\Omega \end{cases}$, the above formula can be rewritten as:

$$= J_0\left[\left(1 + \frac{\delta}{2}\Omega\right) + \frac{\delta}{2}\Omega\right] - J_0\left[\left(1 + \frac{\delta}{2}\Omega\right) - \frac{\delta}{2}\Omega\right] \tag{S14}$$

$$= J_0(A+B) - J_0(A-B) \tag{S15}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{n!n!}\left[\left(\frac{A+B}{2}\right)^{2n} - \left(\frac{A-B}{2}\right)^{2n}\right] \tag{S16}$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{n!n!}\left\{\sum_{k=0}^{2n}\left[C_{2n}^{k}\left(\frac{A}{2}\right)^{2n-k}\left(\frac{B}{2}\right)^{k} - C_{2n}^{k}\left(\frac{A}{2}\right)^{2n-k}\left(\frac{-B}{2}\right)^{k}\right]\right\} \tag{S17}$$

$$= 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n!n!}\left\{\sum_{k=0}^{n-1}\left[C_{2n}^{2k+1}\left(\frac{A}{2}\right)^{2n-2k-1}\left(\frac{B}{2}\right)^{2k+1}\right]\right\} \tag{S18}$$

$$= 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n!n!}\left\{\sum_{k=0}^{n-1}\left[\frac{2n}{2k+1}C_{2n-1}^{2k}\left(\frac{A}{2}\right)^{2n-2k-1}\left(\frac{B}{2}\right)^{2k+1}\right]\right\} \tag{S19}$$

$$= -2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(n+1)!}\left(\frac{A}{2}\right)^{2n+1}\sum_{k=0}^{\infty} b_{n,k}B^{2k+1} \tag{S20}$$

where,

$$[b_{n,k}] = \begin{bmatrix} 1 & -\frac{1}{8} & \frac{1}{192} & \cdots & \frac{(-1)^k}{k!(k+1)!}\left(\frac{1}{2}\right)^{2k} \\ 1 & -\frac{5}{36} & \frac{7}{1125} & \cdots & b_{1,k} \\ 1 & -\frac{3}{20} & \frac{21}{3200} & \cdots & b_{2,k} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & -\frac{1}{6} & \frac{1}{120} & \cdots & \frac{(-1)^k}{(2k+1)!} \end{bmatrix} \tag{S21}$$

The previous formula(S20) can be simplified:

$$-2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(n+1)!}\left(\frac{A}{2}\right)^{2n+1}\sum_{k=0}^{\infty} b_{n,k}B^{2k+1} \tag{S22}$$

$$= -2\sum_{n=0}^{\infty}\left\{\frac{(-1)^n}{n!(n+1)!}\left(\frac{A}{2}\right)^{2n+1}\left[\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}B^{2k+1} + o(B^3)\right]\right\} \tag{S23}$$

Because $|\delta| < 1$,the cubic term $o(B^3)$ can be neglected, thus:

$$\approx -2\sum_{n=0}^{\infty}\left\{\frac{(-1)^n}{n!(n+1)!}\left(\frac{A}{2}\right)^{2n+1}\left[\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}B^{2k+1}\right]\right\} \tag{S24}$$

$$= -2J_1(A)\sin(B) \tag{S25}$$

$$= -2J_1\left[\left(1+\frac{\delta}{2}\right)\Omega\right]\sin\frac{(\delta\Omega)}{2} \tag{S26}$$

Simulation：comparison of original value and approximate value

Fig.S1 shows the error between original and approximate amplitude modulation functions. It can be seen from fig. S1 that, when the coefficient is small, the error is close to zero; although the error increases as the coefficient increases, the error is still small when the coefficient is 1. Therefore, it can be concluded that the approximate function agrees well with the original function in the case of $\delta < 1$.

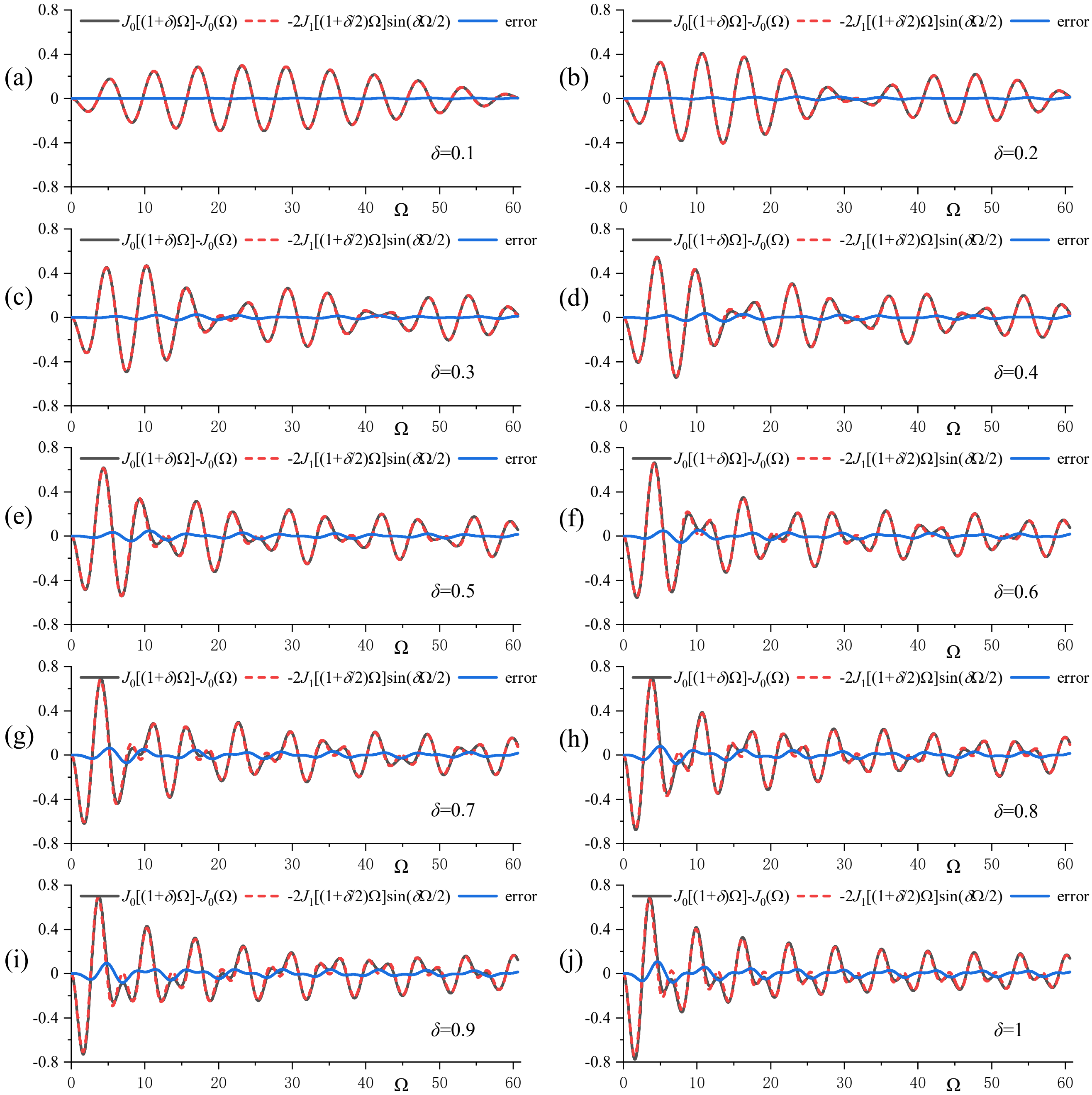

Figure. S1 Comparison of amplitude modulation function with different coefficients. (a) to (j) are the coefficients 0.1 to 1 respectively.