Supplemental Document

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Probing the mode-locking pattern in the parameter space of a Figure-9 laser: supplement

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The Jones Matrices used in this paper are listed in Table S1. The polarization direction corresponding to the transmission of the PBC is defined as the x-axis direction. The electric field after one roundtrip $\vec{E}_{intra}^{rt}(\theta_q,\theta_h,\Delta\varphi_{nl})$ can be expressed as

$$\vec{E}_{intra}^{rt}(\theta_q,\theta_h,\Delta\varphi_{nl}) = M_{PBC,trans}M_{\frac{\lambda}{4}}(\theta_q)M_F\left(\frac{\pi}{4}\right)M_{\frac{\lambda}{2}}(\theta_h)M_{loop}M_{nl}(\Delta\varphi_{nl})M_{\frac{\lambda}{2}}(\theta_h)M_F\left(\frac{\pi}{4}\right)M_{\frac{\lambda}{4}}(\theta_q)\vec{e}_x, \quad \text{(S1)}$$

while $\vec{e}_x = \binom{1}{0}$ is defined as the normalized light electric field vector transmitted by the PBC. The cavity roundtrip transmission T can be expressed as

$$T(\theta_q,\theta_h,\Delta\varphi_{nl}) = |\vec{E}_{intra}^{rt}|^2. \quad \text{(S2)}$$

So T is a function of $\Delta\varphi_{nl}$. The function curve of the T with the $\Delta\varphi_{nl}$ varying between -π to π is defined as the cavity transmission function.

As the splitting ratio varies as a function of the position of the waveplates, the normalized field vector at the entrance of the loop should be expressed as

$$\vec{E}_{loop\ entrance} = M_{\frac{\lambda}{2}}(\theta_h)M_F\left(\frac{\pi}{4}\right)M_{\frac{\lambda}{4}}(\theta_q)\vec{e}_x. \quad \text{(S3)}$$

And the fractions of light respectively split into cw and ccw directions can be calculated by

$$k_1 = |E_{loop\ entrance,x}|^2, k_2 = |E_{loop\ entrance,y}|^2, \quad \text{(S4)}$$

where k_1 and k_2 are the ratios of light intensities for cw and ccw fields. The splitting ratio is defined by k_1/k_2.

Table S1. Jones Matrices for elements in experimental setup

Component	Jones matrix
Half waveplate	$M_{\frac{\lambda}{2}}(\theta) = e^{-\frac{i\pi}{2}}\begin{pmatrix} cos^2\theta - sin^2\theta & 2cos\theta sin\theta \\ 2cos\theta sin\theta & sin^2\theta - cos^2\theta \end{pmatrix}$
Quarter waveplate	$M_{\frac{\lambda}{4}}(\theta) = e^{-\frac{i\pi}{4}}\begin{pmatrix} cos^2\theta + isin^2\theta & (1-i)cos\theta sin\theta \\ (1-i)cos\theta sin\theta & sin^2\theta + icos^2\theta \end{pmatrix}$
Faraday rotator	$M_F(\theta) = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$
PBC (transmission/reflection)	$M_{PBC,trans} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $M_{PBC,refl} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
Mirror	$M_{mirror} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$
Fiber loop	$M_{loop} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
NPSD	$M_{nl}(\Delta\varphi_{nl}) = \begin{pmatrix} e^{i\Delta\varphi_{nl}} & 0 \\ 0 & 1 \end{pmatrix}$

The Fig. S1 shows the typical scope waveforms corresponding to mode-locking, Q-switching, and continuous wave state, respectively.

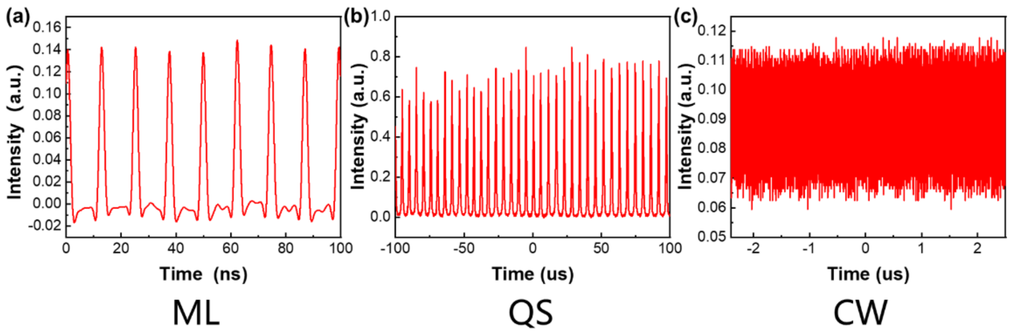

Fig. S1. Typical waveforms corresponding to (a) mode-locking, (b) Q-switching, and (c) continuous wave state, respectively.

Fig. S2 shows the typical mode-locking states obtained in cavity with high symmetry, at a pump power of 300 mW.

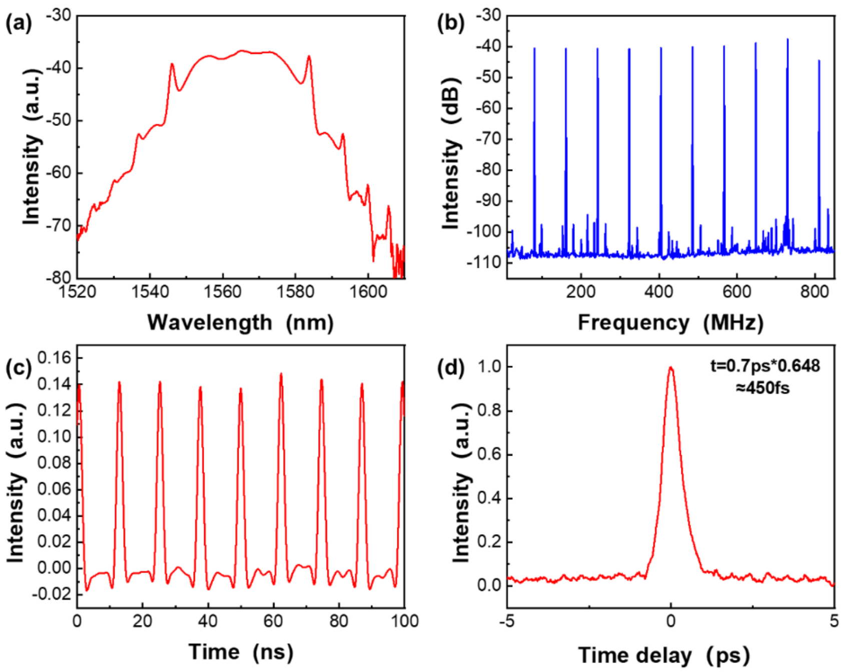

Fig. S2. Typical mode-locking state for the symmetrical cavity. (a) Output spectra. (b) RF spectrum. (c) Output pulse train. (d) Autocorrelation trace.

Fig. S3 shows the spectra curves obtained with pump power of 400 mW, 300 mW, and 200 mW, respectively. These images display the evolution process from muti-solitons bound state to single soliton pulse.

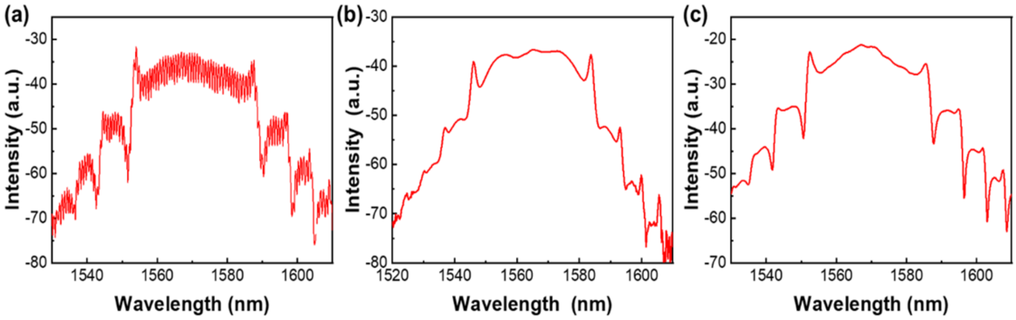

Fig. S3. Spectra obtained with pump power of (a) 400 mW, (c) 300 mW, and (d) 200 mW, respectively.

Fig. S4 shows the typical mode-locking states obtained in cavity with reduced symmetry, at a pump power of 150 mW.

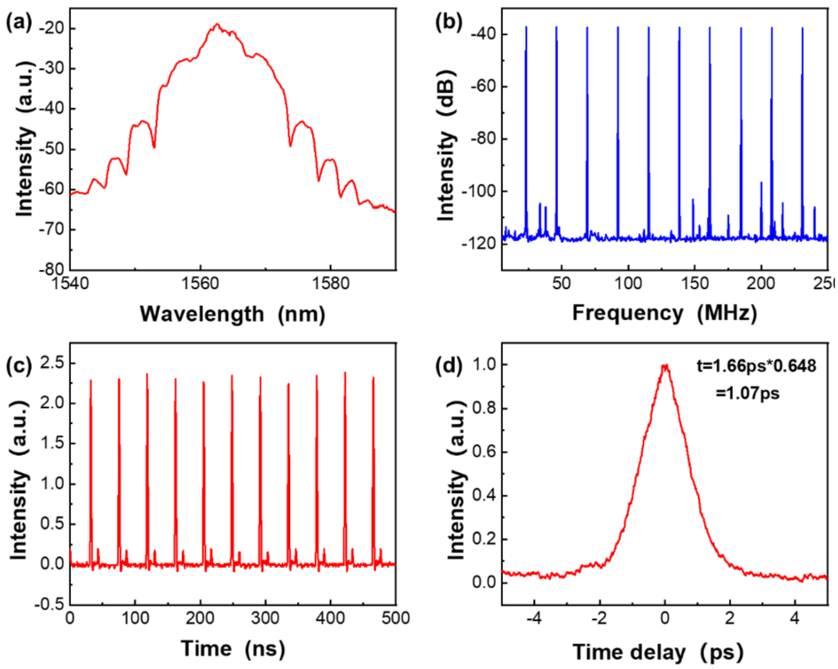

Fig. S4. Typical mode-locking state for the asymmetrical cavity. (a) Output spectra. (b) RF spectrum. (c) Output pulse train. (d) Autocorrelation trace.