Supplemental Document

Optics EXPRESS

Entangling three identical particles via spatial overlap: supplement

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Supplement DOI: https://doi.org/10.6084/m9.figshare.20383386

Parent Article DOI: https://doi.org/10.1364/OE.460866

1. THE COMPUTATIONS OF TRIPARTITE NO-BUNCHING STATES

A. GHZ class

By inserting the entries of T^{GHZ} and S^{GHZ} into Eq. (4) of the main content, we obtain

$$\begin{aligned}\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_3^\dagger|vac\rangle &= (\sum_{j=1}^{3} T_{1j}^{GHZ}\hat{b}_{1j}^\dagger)(\sum_{k=1}^{3} T_{2k}^{GHZ}\hat{b}_{2k}^\dagger)(\sum_{l=1}^{3} T_{3l}^{GHZ}\hat{b}_{3l}^\dagger)|vac\rangle \\ &= (\alpha_1\hat{b}_{11}^\dagger + \alpha_2\hat{b}_{12}^\dagger)(\beta_1\hat{b}_{22}^\dagger + \beta_2\hat{b}_{23}^\dagger)(\gamma_1\hat{b}_{31}^\dagger + \gamma_2\hat{b}_{33}^\dagger)|vac\rangle \\ &= (\alpha_1|1,\downarrow,d_1\rangle + \alpha_2|2,\uparrow,d_1\rangle)(\beta_1|2,\downarrow,d_2\rangle + \beta_3|3,\uparrow,d_2\rangle)(\gamma_1|1,\uparrow,d_3\rangle + \gamma_3|3,\downarrow,d_3\rangle).\end{aligned} \tag{S1}$$

With the postselection of no-bunching states, the unnormalized relevant state $|\Psi_{GHZ}\rangle$ is given by

$$\begin{aligned}|\Psi_{GHZ}\rangle &= \alpha_1\beta_2\gamma_3|1,\downarrow,d_1\rangle|2,\downarrow,d_2\rangle|3,\downarrow,d_3\rangle + \alpha_2\beta_3\gamma_1|2,\uparrow,d_1\rangle|3,\uparrow,d_2\rangle|1,\uparrow,d_3\rangle \\ &= \alpha_1\beta_2\gamma_3|\downarrow d_1\rangle|\downarrow d_2\rangle|\downarrow d_3\rangle + \alpha_2\beta_3\gamma_1|\uparrow d_3\rangle|\uparrow d_1\rangle|\uparrow d_2\rangle,\end{aligned} \tag{S2}$$

which is Eq. (5) of the main content. In the second equality of the above equation, we omit the spatial mode state and assume it to be denoted by the state order. For $|\alpha_i| = |\beta_i| = |\gamma_i| = 1/\sqrt{2}$, the success probability of a tripartite GHZ state becomes $P_{\mathrm{GHZ}} = |\alpha_1\beta_2\gamma_3|^2 + |\alpha_2\beta_3\gamma_1|^2$=1/4.

B. W class

The W class entanglement of identical particles can be obtained without transforming the internal states. We set $(s_1, s_2, s_3) = (\downarrow, \downarrow, \uparrow)$ and insert the transformation operator in Eq. (6) of the main text into Eq. (4). Then, the three-particle transformation relation is given by

$$\begin{aligned}\hat{a}_1^\dagger\hat{a}_2^\dagger\hat{a}_3^\dagger|vac\rangle &= (\sum_{j=1}^{3} T_{1j}^{W}\hat{b}_{1j}^\dagger)(\sum_{k=1}^{3} T_{2k}^{W}\hat{b}_{2k}^\dagger)(\sum_{l=1}^{3} T_{3l}^{W}\hat{b}_{3l}^\dagger)|vac\rangle \\ &= (\alpha_1\hat{b}_{11}^\dagger + \alpha_2\hat{b}_{12}^\dagger + \alpha_3\hat{b}_{13}^\dagger)(\beta_1\hat{b}_{21}^\dagger + \beta_2\hat{b}_{22}^\dagger + \beta_3\hat{b}_{23}^\dagger)(\gamma_1\hat{b}_{31}^\dagger + \gamma_2\hat{b}_{32}^\dagger + \gamma_3\hat{b}_{33}^\dagger)|vac\rangle \\ &= (\alpha_1|1,\downarrow,d_1\rangle + \alpha_2|2,\downarrow,d_1\rangle + \alpha_3|3,\downarrow,d_1\rangle)(\beta_1|1,\downarrow,d_2\rangle + \beta_2|2,\downarrow,d_2\rangle + \beta_3|3,\downarrow,d_2\rangle) \\ &\quad \times (\gamma_1|1,\uparrow,d_3\rangle + \gamma_2|2,\uparrow,d_3\rangle + \gamma_3|3,\uparrow,d_3\rangle).\end{aligned} \tag{S3}$$

After the postselection, the unnormalized relevant state is given by

$$\begin{aligned}|\Psi_W\rangle = \quad & \alpha_1\beta_2\gamma_3|\downarrow,d_1\rangle|\downarrow,d_2\rangle|\uparrow,d_3\rangle + \alpha_1\beta_3\gamma_2|\downarrow,d_1\rangle|\uparrow,d_3\rangle|\downarrow,d_2\rangle \\ & +\alpha_2\beta_1\gamma_3|\downarrow,d_2\rangle|\downarrow,d_1\rangle|\uparrow,d_3\rangle + \alpha_2\beta_3\gamma_1|\uparrow,d_3\rangle|\downarrow,d_1\rangle|\downarrow,d_2\rangle \\ & +\alpha_3\beta_1\gamma_2|\downarrow,d_2\rangle|\uparrow,d_3\rangle|\downarrow,d_1\rangle + \alpha_3\beta_2\gamma_1|\uparrow,d_3\rangle|\downarrow,d_2\rangle|\downarrow,d_1\rangle,\end{aligned} \tag{S4}$$

where the state orders denote the spatial modes. This is Eq. (9) of the main content. For $|\alpha_i| = |\beta_i| = |\gamma_i| = 1/\sqrt{3}$, the success probability of a tripartite W state becomes $P_{\mathrm{W}} = 2/9$.

2. THE MEASURABLE DENSITY MATRIX OF W STATES ACCORDING TO THE DISTINGUISHABILITY CHANGE

In this section, we compute the density matrices of W class that can change according to the particle distinguishability, i.e., **Case I** $\sim$ **IV**. Since **Case I** is trivial to obtain, here we discuss the computations for **Case II, III** and **IV**.

Case II. We can set the distinguishability as $(|d_1\rangle, |d_2\rangle, |d_3\rangle) = (|d_x\rangle, |d_y\rangle, |d_x\rangle)$ $(\langle d_x|d_y\rangle = 0)$ without loss of generality. Then, Eq. (S4) becomes

$$\begin{aligned}|\Psi_W^{\mathrm{II}}\rangle =&(\alpha_1\beta_2\gamma_3|\downarrow\downarrow\uparrow\rangle + \alpha_3\beta_2\gamma_1|\uparrow\downarrow\downarrow\rangle)\otimes|d_xd_yd_x\rangle + (\alpha_1\beta_3\gamma_2|\downarrow\uparrow\downarrow\rangle + \alpha_2\beta_3\gamma_1|\uparrow\downarrow\downarrow\rangle)\otimes|d_xd_xd_y\rangle\\ &+ (\alpha_2\beta_1\gamma_3|\downarrow\downarrow\uparrow\rangle + \alpha_3\beta_1\gamma_2|\downarrow\uparrow\downarrow\rangle)\otimes|d_yd_xd_x\rangle \\ \equiv&|\Psi_{13|2}^{\mathrm{II}}\rangle\otimes|d_xd_yd_x\rangle + |\Psi_{12|3}^{\mathrm{II}}\rangle\otimes|d_xd_xd_y\rangle + |\Psi_{23|1}^{\mathrm{II}}\rangle\otimes|d_yd_xd_x\rangle.\end{aligned} \tag{S5}$$

Note that $|\Psi_{13|2}^{II}\rangle = \alpha_1\beta_2\gamma_3|\downarrow\downarrow\uparrow\rangle + \alpha_3\beta_2\gamma_1|\uparrow\downarrow\downarrow\rangle$ is a bi-separable pure state between subsystem 2 and the others, etc. We obtain the measurable density matrix ρ_W^{II}, Eq. (11) of the main content, by tracing out the distinguishability.

Case III. The particle with internal state $|\uparrow\rangle$ is distinguishable, i.e., $(|d_1\rangle, |d_2\rangle, |d_3\rangle) = (|d_x\rangle, |d_x\rangle, |d_y\rangle)$. Now Eq. (S4) becomes

$$\begin{aligned}|\Psi_W^{\mathrm{III}}\rangle =&(\alpha_1\beta_2\gamma_3 + \alpha_2\beta_1\gamma_3)|\downarrow\downarrow\uparrow\rangle\otimes|d_xd_xd_y\rangle + (\alpha_1\beta_3\gamma_2 + \alpha_3\beta_1\gamma_2)|\downarrow\uparrow\downarrow\rangle\otimes|d_xd_yd_x\rangle\\ &+ (\alpha_2\beta_3\gamma_1 + \alpha_3\beta_2\gamma_1)|\uparrow\downarrow\downarrow\rangle\otimes|d_yd_xd_x\rangle\end{aligned} \tag{S6}$$

and the measurable density matrix ρ_W^{III} after tracing out the distinguishablity is given by

$$\begin{aligned}\rho_W^{\mathrm{III}} =&|\alpha_1\beta_2\gamma_3 + \alpha_2\beta_1\gamma_3|^2|\downarrow\downarrow\uparrow\rangle\langle\downarrow\downarrow\uparrow| \\ &+|\alpha_1\beta_3\gamma_2 + \alpha_3\beta_1\gamma_2|^2|\downarrow\uparrow\downarrow\rangle\langle\downarrow\uparrow\downarrow| + |\alpha_2\beta_3\gamma_1 + \alpha_3\beta_2\gamma_1|^2|\uparrow\downarrow\downarrow\rangle\langle\uparrow\downarrow\downarrow|.\end{aligned} \tag{S7}$$

We see that this state is a statistical mixture of $|\uparrow\uparrow\downarrow\rangle$, $|\uparrow\downarrow\uparrow\rangle$, and $|\downarrow\uparrow\uparrow\rangle$ states, and fully separable.

Case IV. All the three particles are distinguishable with each other, i.e., $(|d_1\rangle, |d_2\rangle, |d_3\rangle) = (|d_x\rangle, |d_y\rangle, |d_z\rangle)$ where $\langle d_x|d_y\rangle = \langle d_y|d_z\rangle = \langle d_x|d_z\rangle = 0$.

Then, the pure state $|\Psi_W^{\mathrm{IV}}\rangle$ and the measurable density matrix ρ_W^{IV} are given by

$$\begin{aligned}|\Psi_W^{\mathrm{IV}}\rangle =&\alpha_1\beta_2\gamma_3|\downarrow\downarrow\uparrow\rangle\otimes|d_xd_yd_z\rangle + \alpha_1\beta_3\gamma_2|\downarrow\uparrow\downarrow\rangle\otimes|d_xd_zd_y\rangle + \alpha_2\beta_1\gamma_3|\downarrow\downarrow\uparrow\rangle\otimes|d_yd_xd_z\rangle\\ &+\alpha_2\beta_3\gamma_1|\uparrow\downarrow\downarrow\rangle\otimes|d_zd_xd_y\rangle + \alpha_3\beta_1\gamma_2|\downarrow\uparrow\downarrow\rangle\otimes|d_yd_zd_x\rangle + \alpha_3\beta_2\gamma_1|\uparrow\downarrow\downarrow\rangle\otimes|d_zd_yd_x\rangle.\end{aligned} \tag{S8}$$

and

$$\begin{aligned}\rho_W^{\mathrm{IV}} =&(|\alpha_1\beta_2\gamma_3|^2 + |\alpha_2\beta_1\gamma_3|^2)|\downarrow\downarrow\uparrow\rangle\langle\downarrow\downarrow\uparrow| + (|\alpha_1\beta_3\gamma_2|^2 \\ &+|\alpha_3\beta_1\gamma_2|^2)|\downarrow\uparrow\downarrow\rangle\langle\downarrow\uparrow\downarrow| + (|\alpha_2\beta_3\gamma_1|^2 + |\alpha_3\beta_2\gamma_1|^2)|\uparrow\downarrow\downarrow\rangle\langle\uparrow\downarrow\downarrow|.\end{aligned} \tag{S9}$$

Except for the postselection probability, ρ_W^{IV} of Eq. (S9) is identical to ρ_W^{III} of Eq. (S7), and thus a fully separable state. This results correspond to Eq. (13) of the main content.

3. DETAILS OF THE EXPERIMENTAL SETUP

A. GHZ class state generation

Figure S1 shows the detailed experimental setup for the GHZ class state generation. Four single-photon states at 780 nm are generated by two type-II non-collinear spontaneous parametric down-conversion (SPDC) setups using two 1 mm long beta-barium borate crystals (BBO1, BBO2), pumped by optical pulses at 390 nm. The 390 nm pumping optical pulses are prepared via second harmonic generation at a lithium triborate (LBO) crystal pumped by 780 nm fs laser pulses having ∼140 fs temporal width and 80 MHz repetition rate. The four photon coincidence count rate is about 2 counts per second with the average pump power of 300 mW. The intrinsic spectral and spatial indistinguishability of the photons are achieved by spectral and spatial mode filtering using 3 nm interference filters (IF), and single-mode optical fibers, respectively. The indistinguishability of the photons is experimentally verified with the high Hong-Ou-Mandel (HOM) interference visibility $V > 0.85$ for all possible photon-pair combinations. Note that the limited HOM visibility is mainly due to the multi-photon contributions of the SPDC sources. Then, one of four photons is used for trigger (T), and the others were sent to the experimental setup via single-mode optical fibers.

Overall, except trigger single photon, we have three single photons, i.e., $a_i^\dagger$, where $i \in \{1,2,3\}$. The detailed state evolution to implement the transition matrix for GHZ state generation is following.

Step 1. By adjusting waveplates H0 and Q0, the polarization state is set as diagonal for all three single photons as

$$|\psi\rangle_{\text{in}} = \frac{1}{2\sqrt{2}}(a^\dagger_{1,H} + a^\dagger_{1,V})(a^\dagger_{2,H} + a^\dagger_{2,V})(a^\dagger_{3,H} + a^\dagger_{3,V})|0\rangle. \tag{S10}$$

Step 2. With PBD0 and half-waveplates (H_S), we have divided the input particles according to the input polarization states and changed the polarization states of each mode to satisfy S^{GHZ} in Eq. (5) in main text. This transition is represented as

$$\begin{aligned} a^\dagger_{1,H} &\to b^\dagger_{2,V}, \quad a^\dagger_{1,V} \to b^\dagger_{1,H},\\ a^\dagger_{2,H} &\to b^\dagger_{3,H}, \quad a^\dagger_{2,V} \to b^\dagger_{1,V},\\ a^\dagger_{3,H} &\to b^\dagger_{3,V}, \quad a^\dagger_{3,V} \to b^\dagger_{2,H}. \end{aligned} \tag{S11}$$

Step 3. The spatial wave functions of photons are overlapped via PBD1–PBD3, then, four-fold coincidence counts among D1–D3 and a trigger detection D_T are taken. The post-selected state is given as

$$|\psi\rangle_{\text{out}} \to \frac{1}{2\sqrt{2}}(b^\dagger_{1,H}b^\dagger_{2,H}b^\dagger_{3,H} + b^\dagger_{1,V}b^\dagger_{2,V}b^\dagger_{3,V}), \tag{S12}$$

which represents a tripartite GHZ state with the success probability of $P_{\text{GHZ}} = \frac{1}{4}$.

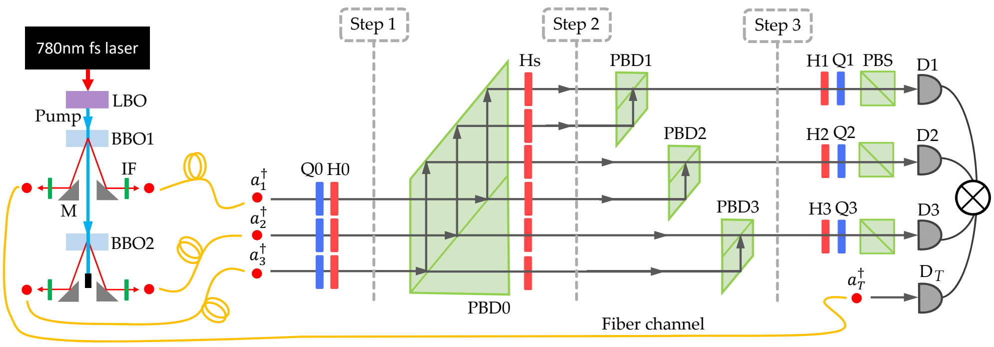

Fig. S1. The experimental setup to generate the GHZ class state. LBO: lithium triborate crystal, BBO: beta-barium borate crystal, M: mirror, IF: interference filter, SMF: single-mode optical fiber, H: half waveplate, Q: quarter waveplate, PBD: polarization beam displacer, PBS: polarizing beamsplitter, D: detector.

B. W class state generation

In order to utilize commercially available a 3×3 multiport or a tritter, we performed the W class state generation experiment using single photons at the telecommunication wavelength. Figure S2 shows the detailed experimental setup for the W class state generation. Four single-photon states at 1556 nm are prepared by collinear SPDC process using two 10 mm long type-II periodically-poled potassium titanyl phosphate (PPKTP) crystals, pumped by 778 nm pulsed laser having ∼170 fs temporal width and 125 MHz repetition rate [1]. With 15 mW of pump power, the four photon coincidence count is achieved about 8 counts per second with superconducting nanowire single-photon detectors having ∼80% of detection efficiency. After the spectral and spatial mode filtering with 3 nm bandpass filters and single-mode optical fibers, we achieved highly indistinguishable photons with the HOM interference visibility of $V > 0.9$ for all possible photon-pair combinations. Then, one of four photons is used for a trigger (T), and the others were sent to the W-state generation experimental setup via single-mode optical fibers.

With the three single photon inputs, the W state generation can be written as below.

Step 1. Using H0 and Q0, the initial three photon input state is prepared as

$$|\psi\rangle_{\text{in}} = a^\dagger_{1,H} a^\dagger_{2,H} a^\dagger_{3,V} |0\rangle. \tag{S13}$$

Setp 2. The transformation operation T^W can be implemented by a symmetrical optical tritter whose transformation is given as [2]

$$\begin{aligned} a^\dagger_{1,H} &\rightarrow \frac{1}{\sqrt{3}}(b^\dagger_{1,H} + b^\dagger_{2,H} + b^\dagger_{3,H}), \\ a^\dagger_{2,H} &\rightarrow \frac{1}{\sqrt{3}}(b^\dagger_{1,H} + e^{\frac{i2\pi}{3}} b^\dagger_{2,H} + e^{-\frac{i2\pi}{3}} b^\dagger_{3,H}) \\ a^\dagger_{3,V} &\rightarrow \frac{1}{\sqrt{3}}(b^\dagger_{1,V} + e^{-\frac{i2\pi}{3}} b^\dagger_{2,V} + e^{\frac{i2\pi}{3}} b^\dagger_{3,V}). \end{aligned} \tag{S14}$$

The post-selection via four-fold coincidences among D1–D3 and D_T provides an output state of

$$|\psi\rangle_{\text{out}} \rightarrow \frac{1}{3\sqrt{3}}(1 + e^{-\frac{i2\pi}{3}}) \left(b^\dagger_{1,H} b^\dagger_{2,H} b^\dagger_{3,V} + b^\dagger_{1,H} b^\dagger_{2,V} b^\dagger_{3,H} + b^\dagger_{1,V} b^\dagger_{2,H} b^\dagger_{3,H} \right) |0\rangle, \tag{S15}$$

that represents a tripartite W state with the success probability of $P_W = \frac{2}{9}$.

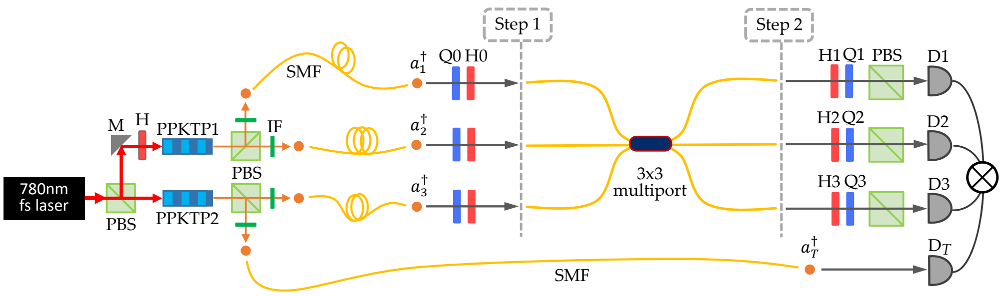

Fig. S2. The experimental setup to generate the W class state. PPKTP: periodically-poled potassium titanyl phosphate crystal, M: mirror, IF: interference filter, SMF: single-mode optical fiber, PBS: polarizing beamsplitter, H: half waveplate, Q: quarter waveplate, D: detector.

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