

## Anti-reflection coatings for epsilon-near-zero materials: supplement

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# Anti-reflection coatings for Epsilon-Near-Zero materials: supplemental document

## 1. IMPLEMENTATION OF THE MULTI-OBJECTIVE GREY WOLF OPTIMISER

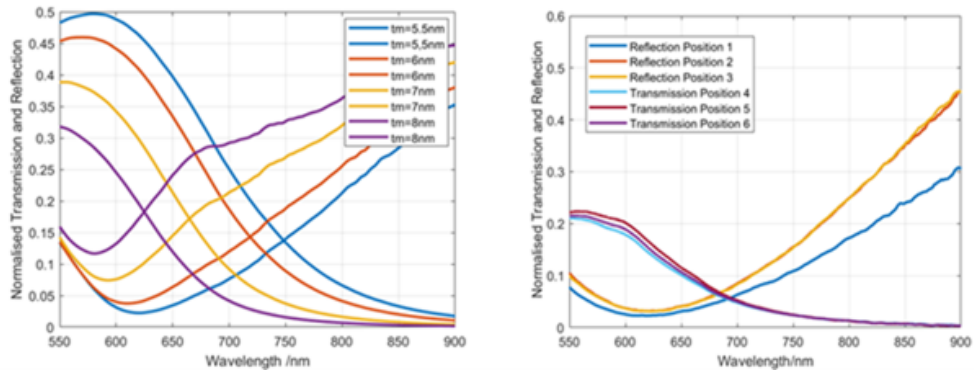
There exist various methods of evaluating multi-objective problems, many like the MOGWO algorithm used here, are meta-heuristic meaning they utilise structures in nature to solve optimisation problems. The MOGWO optimiser is based on the popular conception of wolf hunting behaviour and leadership. A set number of wolves,  $N$ , are generated, each of these wolves has a vector describing its location in the parameter space. The three wolves are chosen out of the population to be, alpha, beta and gamma, these candidates fulfil the FOMs described the best out of the population and due to the selection probability definition there is a pressure to choose candidates from less crowded segments of the parameter space. The positions of the other wolves are updated towards the three wolves that have been selected. The update of the vectors describing each wolf are controlled by a set of randomly generated parameters the  $A$  and  $C$  coefficient vectors at each iteration of the algorithm. The optimiser then stores the non-dominated Pareto optimal solutions in an archive at the end of each iteration. If solutions in the archive are dominated by new solutions they will be removed from the archive, if neither dominate, no solution is added to the archive. When the archive is full the segmentation of the objective space is rearranged and a solution deleted from the most crowded area of the parameter space.

To solve an optimisation problem effectively one must strike a balance between exploration and exploitation. Whether the MOGWO optimiser explores or exploits depends on whether coefficient  $A$  is larger or smaller than one, which is in turn related to the number of iterations which have been executed. The longer the optimiser runs the greater pressure to converge. Vector  $C$  is another variable of importance, this vector generates random values between 0 and 2 in which random weights for prey are provided, this emphasizes or deemphasizes the effect of prey and unlike vector  $A$  this variable is not related in anyway to the number of iterations executed ensuring that there is significant random exploration even as we tend to larger iteration numbers. Grey-wolf optimisers distinguish themselves from the pack of swarm intelligence algorithms by using the three best solutions to lead the search while the others use only one solution. This factor helps to maintain diversity and avoid convergence to local minima.

When running the MOGWO there are several parameters which can be edited by the user; firstly the grid inflation parameter. This parameter determines the amount by which the generated hypercubes are expanded upon by multiplying the maximum variance in each given FOM by the expansion parameter. The other parameter that can be altered is the leader selection pressure, this effects the probability of whether a candidate in a crowded segment/hypercube will be selected as alpha, beta or gamma. The optimisation run from which we take our final solution, i.e. the one we fabricated, utilised a grid inflation parameter of 0.1 and a leader selection pressure of 4.

## 2. THICKNESS NON-UNIFORMITY AND ITS EFFECT ON METAMATERIAL RESPONSE

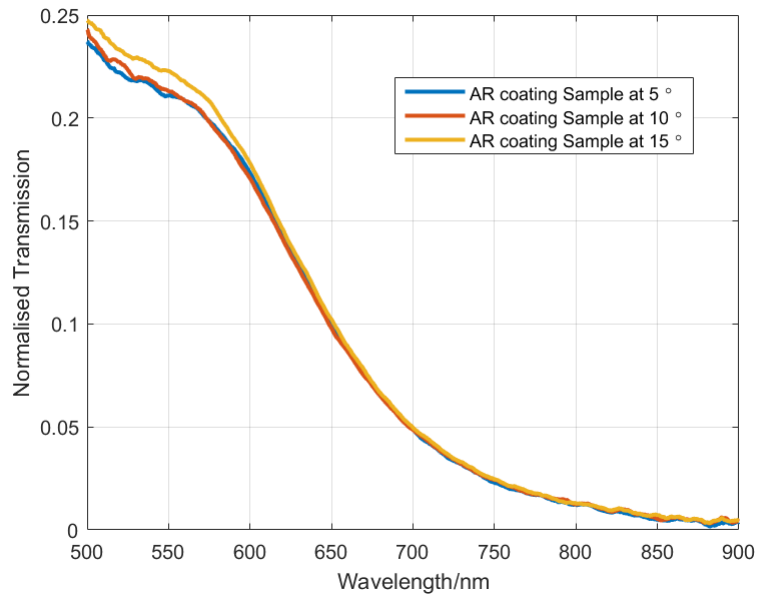
Several areas of the multilayer were characterised in reflection and transmission. It was found that there was some variation in thickness across the sample. We explored how changing the thickness of the silver layers affected the outcome given by the TMM, to provide a comparison for the experimental data. The thicknesses of the dielectric layers were inferred from SEM imaging as a means of accounting for the variations in thickness though the stack. The results are as found in figure [S1](#).



**Fig. S1.** Left: Reflection from AR coated multilayer measured at various points along the sample, Right: Transmission through AR coated multilayer measured at various points on the sample

### 3. METAMATERIAL RESPONSE DEPENDENCE ON THE ANGLE INCIDENCE

Measurements of the transmission were recorded at various angles to ascertain whether the spectral behaviour changed with the angle of incidence. The resulting variation in transmission can be found in figure S2. The variation in transmission with angle was found to be minimal, meaning that the AR coating design is resilient to misalignment in its application.



**Fig. S2.** Metamaterial response at various angles of incidence

### REFERENCES