Supplemental Document

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S1. Optical model of absorptive system.

In this section, we present an alternative derivation of the FSPA. We first consider a simple case where the detection system contains only one absorptive element, and then generalize the result to cases where the detection system contains multiple stacked absorptive elements.

As an example for the simple case where the detection system contains only one absorptive element, we consider the case of a fiber-coupled SNSPD, which is modeled by a birefringent element (corresponding to the birefringent delivery fiber) followed by an absorptive element (corresponding to the meandered nanowire). Referring to Fig. S1, if an incident photon is prepared with a linear vertical polarization (illustrated by the red arrow) at the input port of the delivery fiber (also the output port of the polarization synthesizer), the polarization state of such a photon at the SNSPD end will be elliptically polarized (illustrated by the red ellipse). Similarly, if an incident photon is prepared with a linear horizontal polarization (illustrated by the blue arrow) at the input port of the delivery fiber (also the output port of the polarization synthesizer), the polarization state of such a photon at the SPSPD end will also be elliptically polarized (illustrated by the blue ellipse). Note that according to the imposed power orthogonal condition, the red and blue ellipses share the same axis orientation but with indices of major and minor being switched.

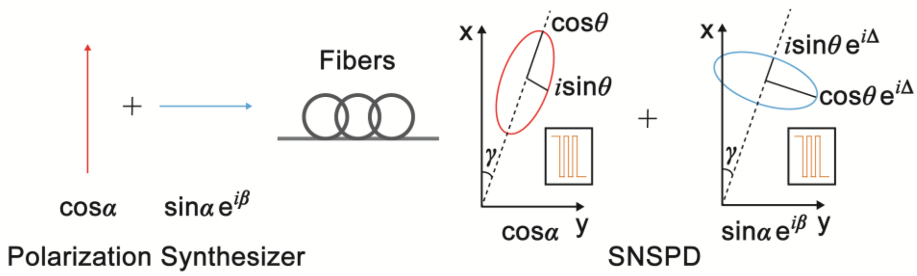

Fig. S1. Theoretical model of the FSPA for simple system with only one absorptive element. The red and blue arrows and ellipses indicate two different cases of orthogonal polarization.

The two polarization ellipses can be defined by three parameters, i.e. γ, θ and Δ, where γ denotes the ellipse's orientation angle with respect to the nanowire of the SNSPD, θ denotes the elliptic axis length and Δ indicates the birefringent phase difference introduced by the delivery fiber. For an incident photon with arbitrary polarization prepared at the polarization synthesizer end with two orthogonal linear polarization components, i.e., $\cos(\alpha)$ and $\sin(\alpha)\exp(i\beta)$, the intensity of the light on the x-axis (parallel to the nanowire) and y-axis (perpendicular to the nanowire) can be calculated straightforwardly as:

$$I_x = \left| \begin{array}{l} (\cos\alpha\cos\theta + i\sin\alpha\sin\theta\exp[i(\Delta+\beta)])\cos\gamma \\ -(i\cos\alpha\sin\theta + \sin\alpha\cos\theta\exp[i(\Delta+\beta)])\sin\gamma \end{array} \right|^2 \tag{S1}$$

$$I_y = \left| \begin{array}{l} (\cos\alpha\cos\theta + i\sin\alpha\sin\theta\exp[i(\Delta+\beta)])\sin\gamma \\ +(i\cos\alpha\sin\theta + \sin\alpha\cos\theta\exp[i(\Delta+\beta)])\cos\gamma \end{array} \right|^2 \tag{S2}$$

Denoting the absorption efficiencies of the SNSPD for light polarized parallelly and perpendicularly to the nanowire by A_{TE} and A_{TM}, respectively, the overall absorption efficiency for the case of arbitrary input polarization reads as:

$$A_{total} = A_{TE} I_x + A_{TM} I_y = A_{total}(A_{TE}, A_{TM}; \gamma, \theta, \Delta; \alpha, \beta) \tag{S3}$$

Eq. (S3) can be regarded as a mathematical function that consists of seven arguments. Among these seven arguments, A_{TE} and A_{TM} are the detector-related parameters that need to be determined. The arguments of γ, θ, and Δ describe the birefringent property of the delivery fiber and the spatial orientation of the nanowire meander. They are unknown and may vary due to environmental perturbations. Finally, the arguments of α and β describe the polarization state of the photon at the end of the polarization synthesizer. They can be controlled at will with instrument programing.

Inspired by the computational scheme [1], we aim to compute A_{TE} and A_{TM} by establishing and then solving a series of combined equations. Similar to the theoretical analysis shown in the main text, we establish a set of equations by choosing $\alpha = 0$, $\beta = 0$ (horizontally polarized), $\alpha = \pi/2$, $\beta = 0$ (vertically polarized), $\alpha = \pi/4$, $\beta = 0$ (45° oblique linearly polarized), and $\alpha = \pi/2$, $\beta = \pi/2$ (circularly polarized). Substituting the above settings into Eq. (S1~S3), we obtain four equations as:

$$\mathrm{A}_{\uparrow} = \mathrm{A}_{\mathrm{TE}}(\cos^2\theta\cos^2\gamma + \sin^2\theta\sin^2\gamma) + \mathrm{A}_{\mathrm{TM}}(\cos^2\theta\cdot\sin^2\gamma + \sin^2\theta\cdot\cos^2\gamma) \tag{S4.1}$$

$$\mathrm{A}_{\rightarrow} = \mathrm{A}_{\mathrm{TE}}(\sin^2\theta\cos^2\gamma + \cos^2\theta\sin^2\gamma) + \mathrm{A}_{\mathrm{TM}}(\sin^2\theta\sin^2\gamma + \cos^2\theta\cos^2\gamma) \tag{S4.2}$$

$$\mathrm{A}_{\nearrow} = \frac{\mathrm{A}_{\mathrm{TE}} + \mathrm{A}_{\mathrm{TM}}}{2} - \frac{\mathrm{A}_{\mathrm{TE}} - \mathrm{A}_{\mathrm{TM}}}{2}\left(\sin 2\gamma\cos\Delta + \cos 2\gamma\sin 2\theta\sin\Delta\right) \tag{S4.3}$$

$$\mathrm{A}_{\circ} = \frac{\mathrm{A}_{\mathrm{TE}} + \mathrm{A}_{\mathrm{TM}}}{2} - \frac{\mathrm{A}_{\mathrm{TE}} - \mathrm{A}_{\mathrm{TM}}}{2}\left(-\sin 2\gamma\sin\Delta + \cos 2\gamma\sin 2\theta\cos\Delta\right) \tag{S4.4}$$

One finds that $\mathrm{A}_{\uparrow} + \mathrm{A}_{\rightarrow} = \mathrm{A}_{\mathrm{TE}} + \mathrm{A}_{\mathrm{TM}}$ and $\mathrm{A}_{\uparrow} - \mathrm{A}_{\rightarrow} = \left(\mathrm{A}_{\mathrm{TE}} - \mathrm{A}_{\mathrm{TM}}\right)\cos 2\gamma\cos 2\theta$. Substituting these relations to Eq. (S4.3-S4.4), it follows that γ, θ and Δ can be eliminated and we have:

$$\begin{cases} \mathrm{A}_{\mathrm{TE}} + \mathrm{A}_{\mathrm{TM}} = \mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow} \\ \left|\mathrm{A}_{\mathrm{TE}} - \mathrm{A}_{\mathrm{TM}}\right| = \sqrt{\left(2\mathrm{A}_{\nearrow} - \mathrm{A}_{\rightarrow} - \mathrm{A}_{\uparrow}\right)^2 + \left(2\mathrm{A}_{\circ} - \mathrm{A}_{\rightarrow} - \mathrm{A}_{\uparrow}\right)^2 + \left(\mathrm{A}_{\rightarrow} - \mathrm{A}_{\uparrow}\right)^2} \end{cases} \tag{S5}$$

Note that Eq. (S5) derived here agrees with Eq. (4) derived in the main text. This completes the alternative derivation for a simple detection system that contains only one absorptive element.

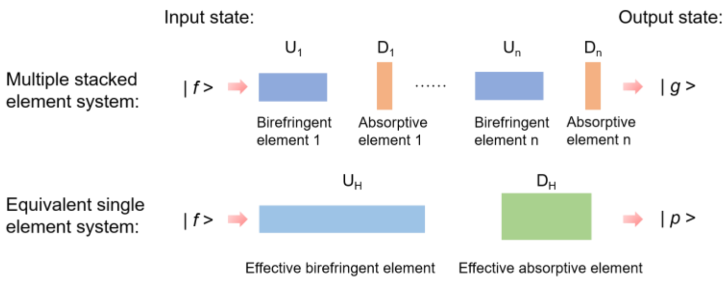

Fig. S2. Schematics of complex optical system with multilayer nanowire gratings.

We next consider a complex detection system that consists of multiple stacked absorptive elements. We assume that theses absorptive elements are connected by birefringent elements, as depicted in Fig. S2. For simplicity, here reflections are neglected and we focus on the transmittance instead of absorptance. Denoting the polarization state of the input photon by $|f\rangle$, the output state then can be written as $|g\rangle = \mathrm{D}_n\mathrm{U}_n\cdots\mathrm{D}_1\mathrm{U}_1|f\rangle$, where D_i are the positive-defined diagonal matrices representing the transmittance of the i_{th} absorptive element and U_i are the unitary polarization manipulation matrices of the i_{th} birefringent element. The overall transmittance of the detection system reads as:

$$\langle g|g\rangle = \langle f|\mathrm{U}_1{}^{+}\mathrm{D}_1\cdots\mathrm{U}_n{}^{+}\mathrm{D}_n\mathrm{D}_n\mathrm{U}_n\cdots\mathrm{D}_1\mathrm{U}_1|f\rangle \tag{S6}$$

In Eq. (S6) the operator $\mathrm{H} = \mathrm{U}_1{}^{+}\mathrm{D}_1\cdots\mathrm{U}_n{}^{+}\mathrm{D}_n\mathrm{D}_n\mathrm{U}_n\cdots\mathrm{D}_1\mathrm{U}_1$ is a Hermitian operator and thus can be unitarily diagonalized, i.e. $\mathrm{H} = \mathrm{U}^{+}{}_{\mathrm{H}}\lambda\,\mathrm{U}_{\mathrm{H}} = \mathrm{U}^{+}{}_{\mathrm{H}}\mathrm{D}_{\mathrm{H}}\mathrm{D}_{\mathrm{H}}\mathrm{U}_{\mathrm{H}}$, where λ is a positive-definite diagonal matrix. From $\langle g|g\rangle = \langle f|\mathrm{H}|f\rangle = \langle f|\mathrm{U}^{+}{}_{\mathrm{H}}\mathrm{D}_{\mathrm{H}}\cdot\mathrm{D}_{\mathrm{H}}\mathrm{U}_{\mathrm{H}}|f\rangle$, we may define $|p\rangle$ = $\mathrm{D}_{\mathrm{H}}\mathrm{U}_{\mathrm{H}}|f\rangle$. As illustrated in Fig. S2, it follows from this definition that in term of transmittance, a lossy transmission system with multiple stacked birefringent and absorptive elements can be effectively regarded as a lossy transmission system with only one birefringent and absorptive element. Therefore, the above result derived for the simple case of one absorptive element, i.e. Eq. (S5), is still applicable for the complex case of multiple stacked absorptive elements. It should be noted that in the latter case, since there is no meaning for A_{TE} and A_{TM}, A_{max} and A_{min} should be used instead.

To validate the conclusion drawn on the stacked absorptive element case, we numerically constructed a two-stack absorptive system where:

$$\mathrm{U}_1 = \begin{bmatrix} \frac{\sqrt{2}}{2}i & -\frac{1}{2}-\frac{1}{2}i \\ \frac{\sqrt{2}}{2}i & \frac{1}{2}+\frac{1}{2}i \end{bmatrix},\ \mathrm{D}_1 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix},$$

$$\mathrm{U}_2 = \begin{bmatrix} \frac{1}{2}+\frac{1}{2}i & -\frac{\sqrt{2}}{2} \\ \frac{1}{2}+\frac{1}{2}i & \frac{\sqrt{2}}{2} \end{bmatrix},\ \mathrm{D}_2 = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.6 \end{bmatrix}.$$

For such a two-stack system, we first calculate the transmittance as a function of the input polarization state using Eq. (S6) by sweeping 10,000 input polarization states on the Poincaré sphere. The results are shown in Fig. S3, where α and β indicate the polarization state, i.e. $|f\rangle$ = $\begin{bmatrix} \cos\alpha \\ \sin\alpha e^{i\beta} \end{bmatrix}$. The maximum and minimum transmittances are then determined from the sweeping results and marked by the green and red dots, i.e. T_{max} = 0.4056 and T_{min} = 0.1150. To make a compare, the maximum and minimum transmittance are also calculated using FSPA, i.e. Eq. (S5). We find that the results obtained by FSPA exactly agree with the results obtained from massive sweeping.

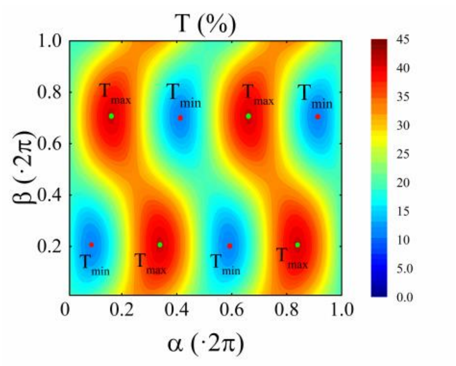

Fig. S3. Numerical simulation of a two-stack lossy transmission system.

S2. Uncertainty analysis on the measurement.

In this section, we provide an estimation on the uncertainty of the measured SDE (σ_{SDE}). According to the SDE discussion in Ref [2,3], the uncertainty of SDE can be generally expressed as: $\sigma_{\mathrm{SDE}} = \sqrt{\sigma_{\mathrm{A}}{}^2 + \sigma_{\mathrm{PR}}{}^2}$, where σ_{A} is the uncertainty of the measured photon count rate, and σ_{PR} is the uncertainty of the measured input photon rate.

The uncertainty of the measured photon count rate σ_{A} is affected by the propagation of the uncertainty of the polarization dependent measurement of $\sigma_{\mathrm{A}i}$, and can be generalized as $\sigma_{\mathrm{A}} = \sqrt{\sum_i \left(\frac{\partial \mathrm{A}}{\partial \mathrm{A}_i}\right)^2 \sigma^2{}_{\mathrm{A}_i}}$. With the help of Eq. (4), we have:

$$
\begin{aligned}
\sigma_{\mathrm{Amax,min}} &= \sqrt{\sum_i \left(\frac{\partial \mathrm{A}_{\mathrm{max,min}}}{\partial \mathrm{A}_i}\right)^2 \sigma^2{}_{\mathrm{A}_i}} \\
&= \left[\left(\frac{1}{2} \pm \frac{\frac{\mathrm{A}_{\rightarrow} - \mathrm{A}_{\uparrow}}{2} + \left(\frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2} - \mathrm{A}_{\nearrow}\right) + \left(\frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2} - \mathrm{A}_{\circ}\right)}{2\sqrt{\left(\frac{\mathrm{A}_{\rightarrow} - \mathrm{A}_{\uparrow}}{2}\right)^2 + \left(\mathrm{A}_{\nearrow} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)^2 + \left(\mathrm{A}_{\circ} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)^2}} \right)^2 \sigma^2{}_{\mathrm{A}_{\rightarrow}} + \right. \\
&\quad \left(\frac{1}{2} \pm \frac{\frac{\mathrm{A}_{\uparrow} - \mathrm{A}_{\rightarrow}}{2} + \left(\frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2} - \mathrm{A}_{\nearrow}\right) + \left(\frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2} - \mathrm{A}_{\circ}\right)}{2\sqrt{\left(\frac{\mathrm{A}_{\rightarrow} - \mathrm{A}_{\uparrow}}{2}\right)^2 + \left(\mathrm{A}_{\nearrow} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)^2 + \left(\mathrm{A}_{\circ} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)^2}} \right)^2 \sigma^2{}_{\mathrm{A}_{\uparrow}} + \\
&\quad \left(\frac{\left(\mathrm{A}_{\nearrow} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)}{\sqrt{\left(\frac{\mathrm{A}_{\rightarrow} - \mathrm{A}_{\uparrow}}{2}\right)^2 + \left(\mathrm{A}_{\nearrow} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)^2 + \left(\mathrm{A}_{\circ} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)^2}} \right)^2 \sigma^2{}_{\mathrm{A}_{\nearrow}} + \\
&\quad \left. \left(\frac{\left(\mathrm{A}_{\circ} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)}{\sqrt{\left(\frac{\mathrm{A}_{\rightarrow} - \mathrm{A}_{\uparrow}}{2}\right)^2 + \left(\mathrm{A}_{\nearrow} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)^2 + \left(\mathrm{A}_{\circ} - \frac{\mathrm{A}_{\rightarrow} + \mathrm{A}_{\uparrow}}{2}\right)^2}} \right)^2 \sigma^2{}_{\mathrm{A}_{\circ}} \right]^{1/2}
\end{aligned} \tag{S7}
$$

Assuming that $\sigma_{\mathrm{A}i}$ are identical for different polarization states, Eq. (S7) is simplified to:

$$\sigma_{\mathrm{Amax,min}} = \sqrt{\frac{\frac{1}{2}+\frac{1}{8}\frac{9\left(A_{\rightarrow}+A_{\uparrow}\right)^2-4A_{\rightarrow}A_{\uparrow}+4\left(A_{\nearrow}+A_{\bigcirc}\right)^2+8\left({A^2}_{\nearrow}+{A^2}_{\bigcirc}\right)-16\left(A_{\rightarrow}+A_{\uparrow}\right)\left(A_{\nearrow}+A_{\bigcirc}\right)}{\left(\frac{A_{\rightarrow}-A_{\uparrow}}{2}\right)^2+\left(A_{\nearrow}-\frac{A_{\rightarrow}+A_{\uparrow}}{2}\right)^2+\left(A_{\bigcirc}-\frac{A_{\rightarrow}+A_{\uparrow}}{2}\right)^2}\pm\frac{A_{\rightarrow}+A_{\uparrow}-A_{\nearrow}-A_{\bigcirc}}{\sqrt{\left(\frac{A_{\rightarrow}-A_{\uparrow}}{2}\right)^2+\left(A_{\nearrow}-\frac{A_{\rightarrow}+A_{\uparrow}}{2}\right)^2+\left(A_{\bigcirc}-\frac{A_{\rightarrow}+A_{\uparrow}}{2}\right)^2}}}{}}\,\sigma_{\bar{\mathrm{A}}} \qquad \text{(S8)}$$

where $\sigma_{\bar{\mathrm{A}}}$ is the mean of the four $\sigma_{\mathrm{A}i}$. Using the estimation of Eq. (s8)and an ensemble of FSPA measurements, the uncertainty of the photon response count rate is determined to be $\sigma_{\mathrm{A,max}}$= ± 0.76% and $\sigma_{\mathrm{A,min}}$= ± 0.98% at 1550 nm.

The uncertainty of the input photon rate is $\sigma_{\mathrm{PR}} = \sqrt{{\sigma_{\mathrm{PM}}}^2 + {\sigma_\lambda}^2 + {\sigma_l}^2 + {\sigma_p}^2 + {\sigma_s}^2 + {\sigma_n}^2}$. At 1550 nm, we have: the power meter term σ_{PM} = ± 3.00%, the wavelength term σ_λ = ± 0.03%, the laser and attenuator term σ_l = ± 0.20%, the polarization synthesizer term σ_p = ± 0.39%, the optical splitter term σ_s = ± 0.03% and the nonlinearity term σ_n = ± 0.2%, resulting σ_{PR} = 3.04%.

Summarizing the above discussion, the overall measurement uncertainty of SDE is finally estimated to be $\sigma_{\mathrm{SDE,max}}$ = ± 3.13% and $\sigma_{\mathrm{SDE,min}}$ = ± 3.19%.

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