Supplemental Document

Optics Letters

Spatiotemporal observation of higher-order modulation instability in a recirculating fiber loop: supplement

FRANÇOIS COPIE,* PIERRE SURET, AND STEPHANE RANDOUX

Univ. Lille, CNRS, UMR 8523 - PhLAM - Physique des Lasers Atomes et Molécules, F-59000 Lille, France
**Corresponding author: francois.copie@univ-lille.fr*

Supplement DOI: https://doi.org/10.6084/m9.figshare.20013341

Parent Article DOI: https://doi.org/10.1364/OL.462389

Spatio-temporal observation of higher-order modulation instability in a recirculating fiber loop: supplemental document

1. CONTINUOUS TRANSITION FROM SEEDED TO SPONTANEOUS MI: EVIDENCES IN THE EVOLUTION OF THE SECOND ORDER MOMENT

In the letter, we report the observation of a gradual loss of coherence in the spatio-temporal evolution of higher-order MI when the frequency detuning between pump and seed lasers decreases down to 0. This is associated to the emergence of spontaneous MI which progressively dominates the dynamics at the expense of higher-order seeded MI. Qualitative signatures of this interplay appear clearly in the single-shot spatio-temporal diagrams of Fig. 2 of the letter. An interesting quantitative insight into this complex interplay is obtained by looking at some statistical indicator and also, by making use of comparisons with numerical simulations. In particular, we focus on the longitudinal evolution of the second-order moment of the optical power $\kappa_4(z)$:

$$\kappa_4(z) = \frac{\langle P(z,t)^2 \rangle}{\langle P(z,t) \rangle^2} \tag{S1}$$

where $P(z,t) = |\Psi(z,t)|^2$ is the optical power and $\langle \cdot \rangle$ denotes average over time t. In Fig. 3 of the letter, we compare the experimental measurements of $\kappa_4(z)$ to numerical simulations of the NLS equation (Equation (1)) for 5 different values of the frequency detuning. Numerical simulations do not include random noise on the initial condition such that the difference between experiments and numerics quantitatively illustrates the influence of spontaneous MI in our experiments.

We have recorded more than 200 single-shot spatio-temporal diagrams by sweeping the frequency detuning Δf in the range ± 13 GHz with a regular step of ~ 125 MHz. The corresponding evolution of the spatio-temporal diagram is available as an animation in Visualisation 1 and 2.

We have performed the computation of $\kappa_4(z)$ for every recording and the result is presented in Fig. S1(a) as a 2D color plot. Each vertical line of this figure represents the longitudinal evolution of κ_4 over 600 km of propagation calculated from one spatio-temporal recording at a given Δf. Bright areas (large κ_4) traduce the existence of a larger number of high power localised structures. Figure S1 clearly reveals a continuous evolution of $\kappa_4(z)$ as a function of Δf which illustrates the interplay between higher-order seeded MI and spontaneous MI as discussed in the letter. Below is a list of noteworthy remarks that can be made:

- The results are almost perfectly symetrical for positive and negative frequency detunings. Discrepancies come from small fluctuations of experimental parameters in the course of the run of acquisitions (~ 2 h);
- For $|\Delta f| > 5$ GHz, well defined bands are observed that translate the quasi-periodic longitudinal dynamics of 1^{st}-order seeded MI;
- For ~ 2 GHz $< |\Delta f| < 5$ GHz, a more complex band structure appears. In particular, 3 bands exist within the first 300 km of propagation associated in the spatio-temporal dynamics to the first maximum compression, coherent splitting in two and three respectively;
- For $|\Delta f| <\sim 2$ GHz, the bands vanish into a unique region where κ_4 is almost uniform when propagation distance is longer than ~ 150 km. It is in this range of Δf that higher-order seeded MI and spontaneous MI exhibit a competition in the spatio-temporal dynamics. In particular, the longitudinal evolution of $\kappa_4(z)$ for $\Delta f \sim 0$ is reminiscent of the one observed in the case of spontaneous MI [5];

For comparison, the same 2D color plot has been constructed from corresponding numerical simulations of the NLS equation without inclusion of noise in the initial condition, and is presented using the same figure layout in Fig. S2. The agreement is remarkable as can be seen from the details within each bands for $\Delta f > 2\,\text{GHz}$ which are also captured in our experiments. Careful comparison eventually reveals that around $\Delta f = 0$, $\kappa_4(z)$ remains equal to 1 in the simulations because no spectral noise in the initial state triggers spontaneous MI. This confirms that the broad uniform region in Fig. S1 around $\Delta f = 0$ find its origin in the emergence of spontaneous MI in the spatio-temporal dynamics of the system.

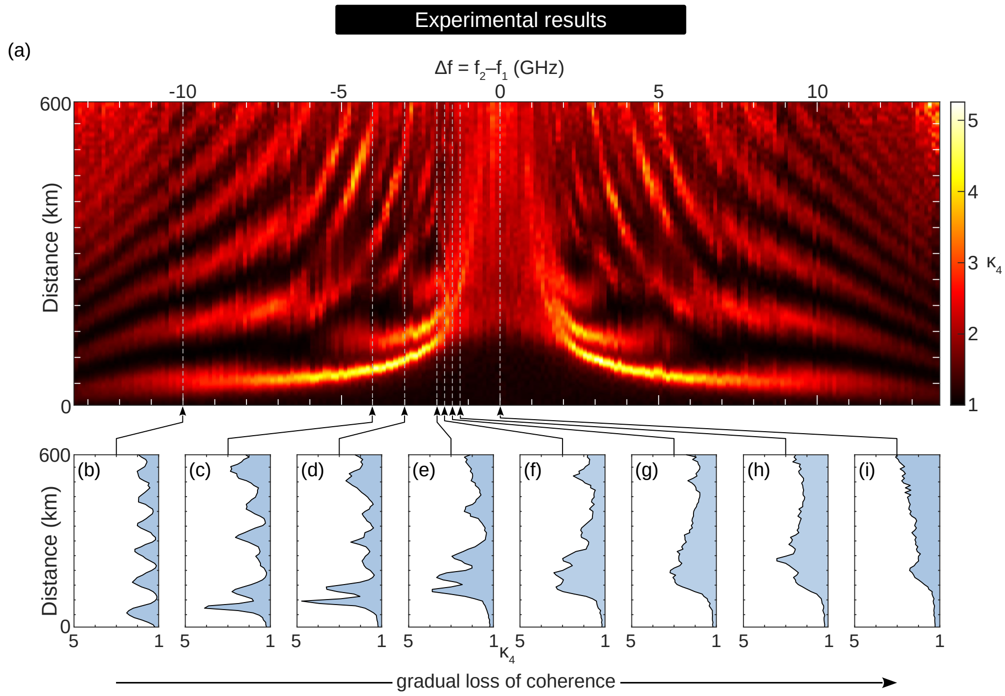

Fig. S1. (a) Longitudinal evolution of the second-order moment $\kappa_4(z)$ as a function of the frequency detuning Δf computed from the experimental recordings. (b-i) $\kappa_4(z)$ curves extracted from (a) for values of Δf indicated by the dashed gray lines.

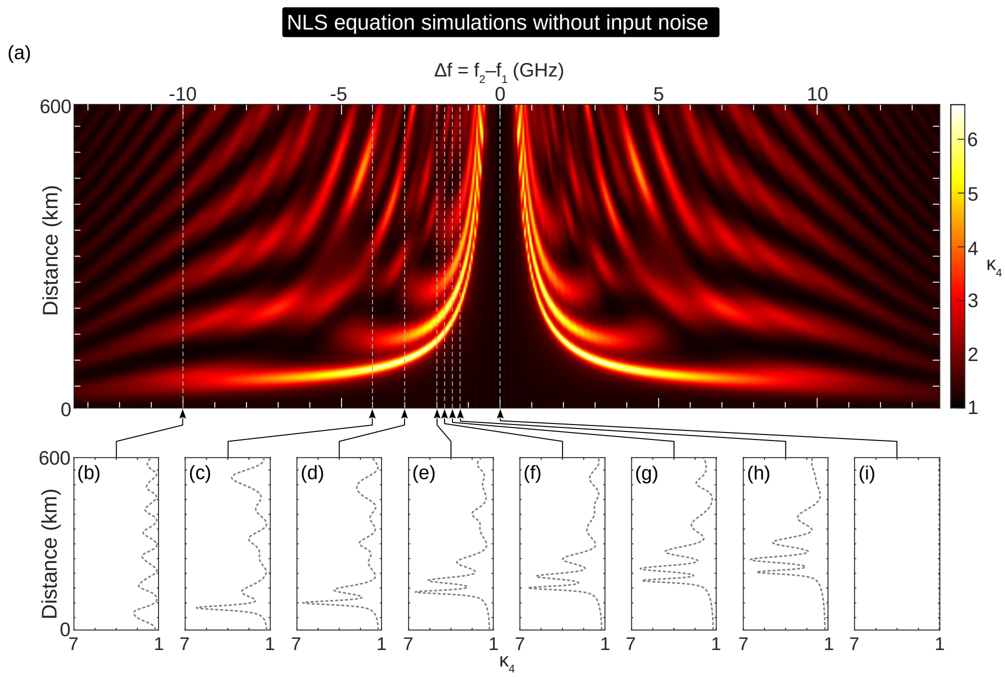

Fig. S2. Same as Fig. S1 with numerical simulations of the NLS equation including realistic losses but no spectral noise in the initial conditions.

2. IMPACT OF THE SINGLE-SIDEBAND EXCITATION AND DISSIPATION ON QUASI-RECURRENCES

In the experiments presented in the letter, MI is seeded by a single spectral sideband located on one side of the pump (i.e. the spectrum of the initial condition is asymetric) which is known as single-sideband excitation. It has been demonstrated that under such condition, FPUT quasi recurrences emerge but are neither in-phase, nor out-of-phase contrary to the symmetric pumping case [1–3]. Indeed, the successive recurrences appear to drift in the spatio-temporal evolution with a given shift that depends on the unstable mode that is excited. If the system additionally features weak dissipation, then after some transient regime the successive recurrences form a regular pattern of π-shifted pulse trains [4]. Traces of this phenomenon are observed in our experiments in the regime of 1$^{\text{st}}$-order seeded MI ($|\Delta f| > 6$ GHz) and are visible in the animations Visualisation 1 and 2. Taking in particular the case $\Delta f \sim -10$ GHz, the first three recurrences are almost in-phase except for a small drift towards longer time that is due to the single-sideband excitation, while the next ones exhibit a larger phase shift that is closer to π.

An in depth experimental investigation of the phenomenon is beyond the scope of the present letter and will be the subject of future work.

REFERENCES

1. P. G. Grinevich and P. M. Santini, "The exact rogue wave recurrence in the NLS periodic setting via matched asymptotic expansions, for 1 and 2 unstable modes," Phys. Lett. A **382**, 973–979 (2018).
2. D. Pierangeli, M. Flammini, L. Zhang, G. Marcucci, A. J. Agranat, P. G. Grinevich, P. M. Santini, C. Conti, and E. DelRe, "Observation of Fermi-Pasta-Ulam-Tsingou Recurrence and Its Exact Dynamics," Phys. Rev. X **8**, 041017 (2018).
3. C. Naveau, P. Szriftgiser, A. Kudlinski, M. Conforti, S. Trillo, and A. Mussot, "Experimental characterization of recurrences and separatrix crossing in modulational instability," Opt. Lett. **44**, 5426 (2019).
4. F. Coppini, P. G. Grinevich, and P. M. Santini, "Effect of a small loss or gain in the periodic nonlinear Schrödinger anomalous wave dynamics," Phys. Rev. E **101**, 032204 (2020).
5. A. E. Kraych, D. Agafontsev, S. Randoux, and P. Suret, "Statistical Properties of the Nonlinear Stage of Modulation Instability in Fiber Optics," Phys. Rev. Lett. **123**, 093902 (2019).