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Transmission and generation of arbitrary W states via an optomechanical interface: supplement

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Transmission and generation of arbitrary W states via an optomechanical interface: supplemental document

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1. QUANTUM JUMP AND CONDITIONAL HAMILTONIAN

The non-Hermitian Hamiltonian of the system is

$$\hat{H}_c = -\sum_{j=0}^{n} \frac{i\kappa_j}{2}\hat{a}_j^\dagger \hat{a}_j - \frac{i\gamma_m}{2}\hat{b}_m^\dagger \hat{b}_m + H.c. \tag{1}$$

where $j = 0, 1, 2, ..., n$, and we assume that $\hbar = 1$, H is the Hamiltonian for the closed system.

The probabilities of an emission from the system at time t and during the time duration Δt are

$$\Delta P_j = \kappa_j \Delta t \langle \psi(t)|\hat{a}_j^\dagger \hat{a}_j|\psi(t)\rangle, \tag{2}$$

$$\Delta P_m = \gamma_m \Delta t \langle \psi(t)|\hat{b}_m^\dagger \hat{b}_m|\psi(t)\rangle, \tag{3}$$

where $j = 0, 1, 2, ..., n$. So, the total probability for the quantum jump happening is $\Delta P_s = \sum_{j=0}^{n} \Delta P_j + \Delta P_m$.

And in fact, in the adiabatic process, the mechanical mode is in dark state, the probability of quantum happens for the mechanical mode will be far less than 1, which can be ignored in the experiment.

If there is an emission from the cavity a_i, the system jumps to the renormalized state

$$\frac{\hat{a}_i|\psi(t)\rangle}{\sqrt{\langle \psi(t)|\hat{a}_i^\dagger \hat{a}_i|\psi(t)\rangle}} \tag{4}$$

If there is an emission from the mechanical mode b_m, the system jumps to the renormalized state

$$\frac{\hat{b}_m|\psi(t)\rangle}{\sqrt{\langle \psi(t)|\hat{b}_m^\dagger \hat{b}_m|\psi(t)\rangle}} \tag{5}$$

If there is no emission, the system will evolve depending on the non-Hermitian Hamiltonian as

$$
\begin{aligned}
& \frac{\exp(-iH_c\Delta t)|\psi(t)\rangle}{\sqrt{\langle\psi(t)|\exp(-iH_c\Delta t)^{\dagger}\exp(-iH_c\Delta t)|\psi(t)\rangle}} \\
\approx\ & \frac{(1-iH_c\Delta t)|\psi(t)\rangle}{\sqrt{\langle\psi(t)|(1-iH_c\Delta t)^{\dagger}(1-iH_c\Delta t)|\psi(t)\rangle}} \\
\approx\ & \frac{(1-\Delta t\sum_{j=0}^{n}\frac{\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j-\Delta t\frac{\gamma_m}{2}\hat{b}_m^{\dagger}\hat{b}_m-i\Delta tH)|\psi(t)\rangle}{\sqrt{\langle\psi(t)|(1-iH_c\Delta t)^{\dagger}(1-iH_c\Delta t)|\psi(t)\rangle}} \\
\approx\ & \frac{(1-\Delta t\sum_{j=0}^{n}\frac{\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j-\Delta t\frac{\gamma_m}{2}\hat{b}_m^{\dagger}\hat{b}_m-i\Delta tH)|\psi(t)\rangle}{\sqrt{1-\Delta t\sum_{j=0}^{n}\langle\psi(t)|\kappa_j\hat{a}_j^{\dagger}\hat{a}_j|\psi(t)\rangle-\Delta t\gamma_m\langle\psi(t)|\hat{b}_m^{\dagger}\hat{b}_m|\psi(t)\rangle}} \\
=\ & \frac{(1-\Delta t\sum_{j=0}^{n}\frac{\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j-\Delta t\frac{\gamma_m}{2}\hat{b}_m^{\dagger}\hat{b}_m-i\Delta tH)|\psi(t)\rangle}{\sqrt{1-\Delta P_s}} \qquad (6)
\end{aligned}
$$

where we have got rid of the high-order terms of Δt.

Then, after the time duration Δt, the density matrix will become

$$
\begin{aligned}
& |\psi(t+\Delta t)\rangle\langle\psi(t+\Delta t)| \\
=\ & \sum_{j=0}^{n}\Delta P_j\frac{\hat{a}_j|\psi(t)\rangle\langle\psi(t)|\hat{a}_j^{\dagger}}{\langle\psi(t)|\hat{a}_j^{\dagger}\hat{a}_j|\psi(t)\rangle}+\Delta P_m\frac{\hat{b}_m|\psi(t)\rangle\langle\psi(t)|\hat{b}_m^{\dagger}}{\langle\psi(t)|\hat{b}_m^{\dagger}\hat{b}_m|\psi(t)\rangle} \qquad (7) \\
+\ & (1-\Delta P_s)\frac{(1-\Delta t\sum_{j=0}^{n}\frac{\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j-\Delta t\frac{\gamma_m}{2}\hat{b}_m^{\dagger}\hat{b}_m-i\Delta tH)|\psi(t)\rangle\langle\psi(t)|(1-\Delta t\sum_{j=0}^{n}\frac{\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j-\Delta t\frac{\gamma_m}{2}\hat{b}_m^{\dagger}\hat{b}_m+i\Delta tH)}{1-\Delta P_s} \\
\approx\ & \Delta t\sum_{j=0}^{n}\kappa_j\hat{a}_j|\psi(t)\rangle\langle\psi(t)|\hat{a}_j^{\dagger}+\Delta t\gamma_m\hat{b}_m|\psi(t)\rangle\langle\psi(t)|\hat{b}_m^{\dagger} \qquad (8) \\
+\ & |\psi(t)\rangle\langle\psi(t)|-i\Delta tH|\psi(t)\rangle\langle\psi(t)|+i\Delta t|\psi(t)\rangle\langle\psi(t)|H-\Delta t\sum_{j=0}^{n}\frac{\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j|\psi(t)\rangle\langle\psi(t)|-\Delta t|\psi(t)\rangle\langle\psi(t)|\sum_{j=0}^{n}\frac{\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j \\
-\ & \Delta t\frac{\gamma_m}{2}\hat{b}_m^{\dagger}\hat{b}_m|\psi(t)\rangle\langle\psi(t)|-\Delta t\frac{\gamma_m}{2}|\psi(t)\rangle\langle\psi(t)|\hat{b}_m^{\dagger}\hat{b}_m
\end{aligned}
$$

The (7) and (8) come from the quantum jump, and the master equation can be written as

$$
\begin{aligned}
\frac{\Delta\rho}{\Delta t} =\ & \sum_{j=0}^{n}\kappa_j\hat{a}_j\rho\hat{a}_j^{\dagger}+\gamma_m\hat{b}_m\rho\hat{b}_m^{\dagger} \qquad (9) \\
+\ & -iH\rho+i\rho H-\sum_{j=0}^{n}\frac{\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j\rho-\sum_{j=0}^{n}\frac{\kappa_j}{2}\rho\hat{a}_j^{\dagger}\hat{a}_j \\
-\ & \frac{\gamma_m}{2}\hat{b}_m^{\dagger}\hat{b}_m\rho-\frac{\gamma_m}{2}\rho\hat{b}_m^{\dagger}\hat{b}_m \\
=\ & -i[H,\rho]+\sum_{j=0}^{n}\frac{\kappa_j}{2}(2\hat{a}_j\rho\hat{a}_j^{\dagger}-\hat{a}_j^{\dagger}\hat{a}_j\rho-\rho\hat{a}_j^{\dagger}\hat{a}_j)+\frac{\gamma_m}{2}(2\hat{b}_m\rho\hat{b}_m^{\dagger}-\hat{b}_m^{\dagger}\hat{b}_m\rho-\rho\hat{b}_m^{\dagger}\hat{b}_m) \qquad (10)
\end{aligned}
$$

In the basis of the bare state, we can calculate the master equation with the rotating wave approximation, then we can replace H with the linearized interaction Hamiltonian $\hat{H}_I$, where $\hat{H}_I=\sum_{j=0}^{n}g_j\left(\hat{a}_j^{\dagger}\hat{b}_m+\hat{b}_m^{\dagger}\hat{a}_j\right)$. So that, if we eliminate the processes in which the quantum jump happened by applying the post selection, the non-Hermitian conditional Hamiltonian can be used to describe the evolution of the system as:

$$
\frac{\Delta\tilde{\rho}}{\Delta t}=-i[\hat{H}_c,\tilde{\rho}] \qquad (11)
$$

Where $\hat{H}_c=-\sum_{j=0}^{n}\frac{i\kappa_j}{2}\hat{a}_j^{\dagger}\hat{a}_j-\frac{i\gamma_m}{2}\hat{b}_m^{\dagger}\hat{b}_m+\hat{H}_I$. In fact, if only considering the coherent decay, the process is a coherent nonunitary evolution, it can also be described by the Schrödinger equation with the conditional Hamiltonian.

2. THE CONDITIONAL HAMILTONIAN IN THE ADIABATIC EVOLUTION

Under the condition of $g_i \gg \kappa_i$ $(i = 1, 2, ..., n)$, for simplicity, we first neglect κ_i and only consider the damping of κ_0 and γ_m. In the basis of $|100...0\rangle_{a_0a_1...a_nb_m}$, $|010...0\rangle_{a_0a_1...a_nb_m}$,..., $|000...1\rangle_{a_0a_1...a_nb_m}$, the conditional Hamiltonian is

$$\hat{H}_c = \begin{bmatrix} -\frac{i\kappa_0}{2} & 0 & \cdots & 0 & g_0 \\ 0 & 0 & \cdots & 0 & g_1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & g_n \\ g_0 & g_1 & \cdots & g_n & -\frac{i\gamma_m}{2} \end{bmatrix} \quad (12)$$

If we transform the basis to $|\phi_1(t)\rangle$, $|\phi_2(t)\rangle$,...,$|\phi_{n+2}(t)\rangle$, where $|\phi_1(t)\rangle$ to $|\phi_n(t)\rangle$ are n dark states and $|\phi_{n+1}(t)\rangle$ and $|\phi_{n+2}(t)\rangle$ are two bright states, the conditional Hamiltonian can be transformed as

$$\hat{\tilde{H}}_c^{[(n+2)\times(n+2)]} = \begin{bmatrix} -\frac{i\kappa_0 g_1^2}{2s_1^2} & -\frac{i\kappa_0 g_0 g_1 g_2}{2s_1^2 s_2} & -\frac{i\kappa_0 g_0 g_1 g_3}{2s_1 s_2 s_3} & \cdots & -\frac{i\kappa_0 g_0 g_1 g_n}{2s_1 s_{n-1} s_n} & \tilde{H}_c^{(1,n+1)} & \tilde{H}_c^{(1,n+2)} \\ -\frac{i\kappa_0 g_0 g_1 g_2}{2s_1^2 s_2} & -\frac{i\kappa_0 g_0^2 g_2^2}{2s_1^2 s_2^2} & -\frac{i\kappa_0 g_0^2 g_2 g_3}{2s_1 s_2^2 s_3} & \cdots & -\frac{i\kappa_0 g_0^2 g_2 g_n}{2s_1 s_2 s_{n-1} s_n} & \tilde{H}_c^{(2,n+1)} & \tilde{H}_c^{(2,n+2)} \\ -\frac{i\kappa_0 g_0 g_1 g_3}{2s_1 s_2 s_3} & -\frac{i\kappa_0 g_0^2 g_2 g_3}{2s_1 s_2^2 s_3} & -\frac{i\kappa_0 g_0^2 g_3^2}{2s_2^2 s_3^2} & \cdots & -\frac{i\kappa_0 g_0^2 g_3 g_n}{2s_2 s_3 s_{n-1} s_n} & \tilde{H}_c^{(3,n+1)} & \tilde{H}_c^{(3,n+2)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{i\kappa_0 g_0 g_1 g_{n-1}}{2s_1 s_{n-2} s_{n-1}} & -\frac{i\kappa_0 g_0^2 g_2 g_{n-1}}{2s_1 s_2 s_{n-2} s_{n-1}} & -\frac{i\kappa_0 g_0^2 g_3 g_{n-1}}{2s_2 s_3 s_{n-2} s_{n-1}} & \cdots & -\frac{i\kappa_0 g_0^2 g_{n-1} g_n}{2s_{n-2} s_{n-1}^2 s_n} & \tilde{H}_c^{(n-1,n+1)} & \tilde{H}_c^{(n-1,n+2)} \\ -\frac{i\kappa_0 g_0 g_1 g_n}{2s_1 s_{n-1} s_n} & -\frac{i\kappa_0 g_0^2 g_2 g_n}{2s_1 s_2 s_{n-1} s_n} & -\frac{i\kappa_0 g_0^2 g_3 g_n}{2s_2 s_3 s_{n-1} s_n} & \cdots & -\frac{i\kappa_0 g_0^2 g_n^2}{2s_{n-1}^2 s_n^2} & \tilde{H}_c^{(n,n+1)} & \tilde{H}_c^{(n,n+2)} \\ \tilde{H}_c^{(n+1,1)} & \tilde{H}_c^{(n+1,2)} & \tilde{H}_c^{(n+1,3)} & \cdots & \tilde{H}_c^{(n+1,n)} & \tilde{H}_c^{(n+1,n+1)} & \tilde{H}_c^{(n+1,n+2)} \\ \tilde{H}_c^{(n+2,1)} & \tilde{H}_c^{(n+2,2)} & \tilde{H}_c^{(n+2,3)} & \cdots & \tilde{H}_c^{(n+2,n)} & \tilde{H}_c^{(n+2,n+1)} & \tilde{H}_c^{(n+2,n+2)} \end{bmatrix} \quad (13)$$

where $(\hat{\tilde{H}}_c^{[(n+2)\times(n+2)]})_{ij} = \langle\phi_i|\hat{H}_c|\phi_j\rangle$.

In the adiabatic process, initially, if the system is in dark state, there will not be excitations with the bright states, so that, the coherent decay evolution of the system will only depend on the first n rows and n columns of the conditional Hamiltonian. We extract the first n rows and n columns of the matrix of $\hat{\tilde{H}}_c$ as

$$\begin{aligned} \hat{\tilde{H}}_c^{(n\times n)} &= \begin{bmatrix} -\frac{i\kappa_0 g_1^2}{2s_1^2} & -\frac{i\kappa_0 g_0 g_1 g_2}{2s_1^2 s_2} & -\frac{i\kappa_0 g_0 g_1 g_3}{2s_1 s_2 s_3} & \cdots & -\frac{i\kappa_0 g_0 g_1 g_n}{2s_1 s_{n-1} s_n} \\ -\frac{i\kappa_0 g_0 g_1 g_2}{2s_1^2 s_2} & -\frac{i\kappa_0 g_0^2 g_2^2}{2s_1^2 s_2^2} & -\frac{i\kappa_0 g_0^2 g_2 g_3}{2s_1 s_2^2 s_3} & \cdots & -\frac{i\kappa_0 g_0^2 g_2 g_n}{2s_1 s_2 s_{n-1} s_n} \\ -\frac{i\kappa_0 g_0 g_1 g_3}{2s_1 s_2 s_3} & -\frac{i\kappa_0 g_0^2 g_2 g_3}{2s_1 s_2^2 s_3} & -\frac{i\kappa_0 g_0^2 g_3^2}{2s_2^2 s_3^2} & \cdots & -\frac{i\kappa_0 g_0^2 g_3 g_n}{2s_2 s_3 s_{n-1} s_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\frac{i\kappa_0 g_0 g_1 g_{n-1}}{2s_1 s_{n-2} s_{n-1}} & -\frac{i\kappa_0 g_0^2 g_2 g_{n-1}}{2s_1 s_2 s_{n-2} s_{n-1}} & -\frac{i\kappa_0 g_0^2 g_3 g_{n-1}}{2s_2 s_3 s_{n-2} s_{n-1}} & \cdots & -\frac{i\kappa_0 g_0^2 g_{n-1} g_n}{2s_{n-2} s_{n-1}^2 s_n} \\ -\frac{i\kappa_0 g_0 g_1 g_n}{2s_1 s_{n-1} s_n} & -\frac{i\kappa_0 g_0^2 g_2 g_n}{2s_1 s_2 s_{n-1} s_n} & -\frac{i\kappa_0 g_0^2 g_3 g_n}{2s_2 s_3 s_{n-1} s_n} & \cdots & -\frac{i\kappa_0 g_0^2 g_n^2}{2s_{n-1}^2 s_n^2} \end{bmatrix} \\ &= -\frac{i\kappa_0}{2} M \end{aligned} \quad (14)$$

where $M = (1 - \frac{g_0^2}{s_n^2})|\phi_0\rangle\langle\phi_0|$ and $|\phi_0\rangle = \frac{1}{\sqrt{1-\frac{g_0^2}{s_n^2}}}[\phi_1^{(1)}, \phi_2^{(1)}, \phi_3^{(1)}, ..., \phi_n^{(1)}]^T$, where $\phi_i^{(1)}$ $(i = 1, 2, ..., n)$ is the first value of $|\phi_i\rangle$.

3. THE CHARACTERISTICS OF V MATRIX

$V(t)$ matrix is defined as $V(t) = \frac{dU^\dagger(t)}{dt}U(t)$ and $\alpha(t) = U^\dagger(t)C(t)$, and there is

$$\frac{d\alpha(t)}{dt} = -[\frac{\kappa_0}{2}\Lambda(t) - V(t)]\alpha(t) \quad (15)$$

Because $\frac{d}{dt}[U^\dagger(t)U(t)] = 0$, so we can get $\frac{dU^\dagger(t)}{dt}U(t) + U^\dagger(t)\frac{dU^\dagger(t)}{dt} = 0$, then there is $V(t) + V^\dagger(t) = 0$. Since that, the matrix $V(t)$ is real, we have $V^\dagger(t) = V^T(t)$, so we can get $V_{ij}(t) + V_{ji}(t) = 0$. If $\kappa_0 = 0$, there is

$$
\begin{aligned}
\frac{d}{dt}\sum_{i=1}^{n}\alpha_i^2(t) &= 2\sum_{i=1}^{n}\alpha_i(t)\frac{d\alpha_i\,(t)}{dt} \\
&= 2\sum_{i=1}^{n}\alpha_i(t)[\sum_{j=1}^{n}V_{ij}(t)\alpha_j(t)] \\
&= 2\sum_{i,j=1}^{n}V_{ij}(t)\alpha_i(t)\alpha_j(t) \\
&= 0 \qquad \textbf{(16)}
\end{aligned}
$$

So that, the function of $V(t)$ is only to redistribute of the populations in every $\varphi_i(t)$.