Supplemental Document

Optics Letters

Zernike-like Laguerre–Gaussian orthonormal polynomials for optical field reconstruction: supplement

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Supplementary Material

I. CENTROID FORMULAE USING LAGUERRE-GAUSSIAN POLYNOMIALS

Suppose that we wish to know the centroid of a real-valued function describing an optical field, such as, for example, the centroid of an intensity distribution $I(r,\theta)$. The function $I(r,\theta)$ may be expressed as a summation of Laguerre-Gaussian polynomials:

$$I(r,\theta) = \sum_{n,m} c_n^m \, F_n^m(r,\theta,\gamma) \,, \tag{S1}$$

where $F_n^m(r,\theta,\gamma)$ is defined in Eq. (13) of the main manuscript and

$$c_n^m = \int_0^{2\pi}\int_0^{\infty} I(r,\theta) F_n^m(r,\theta,\gamma) \, r \, dr \, d\theta \,. \tag{S2}$$

The centroid $(\bar{x},\bar{y})$ is given by

$$\begin{aligned} \bar{x} &= \frac{\int_0^{2\pi}\int_0^{\infty} I(r,\theta) r^2 \cos\theta \, dr \, d\theta}{\int_0^{2\pi}\int_0^{\infty} I(r,\theta) r \, dr \, d\theta} \\ \bar{y} &= \frac{\int_0^{2\pi}\int_0^{\infty} I(r,\theta) r^2 \sin\theta \, dr \, d\theta}{\int_0^{2\pi}\int_0^{\infty} I(r,\theta) r \, dr \, d\theta} \end{aligned} \,. \tag{S3}$$

Taking account of the definitions in Eqs. (14) of the main manuscript and Eq. (S1) above, this may be written as

$$\begin{aligned} \bar{x} &= \frac{C_x}{C_0} \\ \bar{y} &= \frac{C_y}{C_0} \end{aligned} \,, \tag{S4}$$

where

$$\begin{aligned} C_x &= \frac{1}{\gamma}\sum_{n=odd}\sum_{s=0}^{(n-1)/2} c_n^1 N_n^1 (-2)^s (s+1) \binom{(n+1)/2}{(n-1)/2-s} \\ C_y &= \frac{1}{\gamma}\sum_{n=odd}\sum_{s=0}^{(n-1)/2} c_n^{-1} N_n^{-1} (-2)^s (s+1) \binom{(n+1)/2}{(n-1)/2-s} \\ C_0 &= \sum_{n=even}\sum_{s=0}^{n/2} c_n^0 N_n^0 (-2)^s \binom{n/2}{n/2-s} \end{aligned} \,. \tag{S5}$$

II. CENTROID FORMULAE USING ELLIPTICAL LAGUERRE-GAUSSIAN POLYNOMIALS

In calculating the centroid of a real-valued function describing an optical field (such as an intensity distribution $I(r,\theta)$) it is convenient to calculate the centroid $(\bar{u},\bar{v})$ in the (u,v) plane and then use Eqs. (15) of the main manuscript to transform into $(\bar{x},\bar{y})$. In this case the function is assumed to be a summation of elliptical Laguerre-Gaussian polynomials:

$$I(r',\theta') = \sum_{n,m} c_n^m \, \mathcal{F}_n^m(r',\theta',\gamma) \, , \tag{S6}$$

where $\mathcal{F}_n^m(r',\theta',\gamma)$ is defined in Eq. (18) of the main manuscript and

$$c_n^m = \int_0^{2\pi}\int_0^{\infty} I(r',\theta') \mathcal{F}_n^m(r',\theta',\gamma) \, r' \, dr' \, d\theta' \, . \tag{S7}$$

Following the argument in Section I, above, the centroid in the (u,v) plane is

$$\begin{aligned} \bar{u} &= \frac{C'_x}{C'_0} \\ \bar{v} &= \frac{C'_y}{C'_0} \end{aligned} \, , \tag{S8}$$

where C'_x, C'_y, and C'_0 are in the form of Eqs. (S5), except in which N_n^m as defined in Eqs. (14) of the main manuscript is replaced with $\mathcal{N}_n^m$ as defined in Eqs. (19) of the main manuscript. The centroid $(\bar{x},\bar{y})$ in the (x,y) plane is then

$$\begin{aligned} \bar{x} &= \bar{u} \cos\phi - \frac{1}{a} \bar{v} \sin\phi \\ \bar{y} &= \bar{u} \sin\phi + \frac{1}{a} \bar{v} \cos\phi \end{aligned} \, . \tag{S9}$$