Supplemental Document

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Anomalous π modes by Floquet engineering in optical lattices with long-range coupling: supplement

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Section I. Zero modes in static case with long-range coupling.

As shown in Fig. 1 in the main text, the long-range coupling result in two pairs of degenerate points (DPs). If the 3rd nearest-neighbor (NN) couplings are dimerized, both DPs could open a nontrivial gap, as shown in Fig. S1(a) (red for gap1 and blue for gap2). It implies more zero modes compared to the traditional Su-Schrieffer-Heeger (SSH) case with bandgap opening only at the boundaries of the momentum space (i.e., gap1). Figure S1(b) shows the mode diagram under open boundary condition (OBC) with N=40. It is found that four zero modes emerge inside the bandgap. The two edge modes marked by red dots correspond to the original zero modes, which feature π phase shifts for the first and the third edge sites [Fig. S1(c)]. While the other two with blue dots are new zero modes with different features induced by the long-range coupling [Fig. S1(d)]. To be mentioned, the topological phases and associated zero modes in static SSH model with long-range coupling have been theoretically analyzed [1,2], which demonstrate richer zero modes characterized by winding number W=-1, 1, and 2.

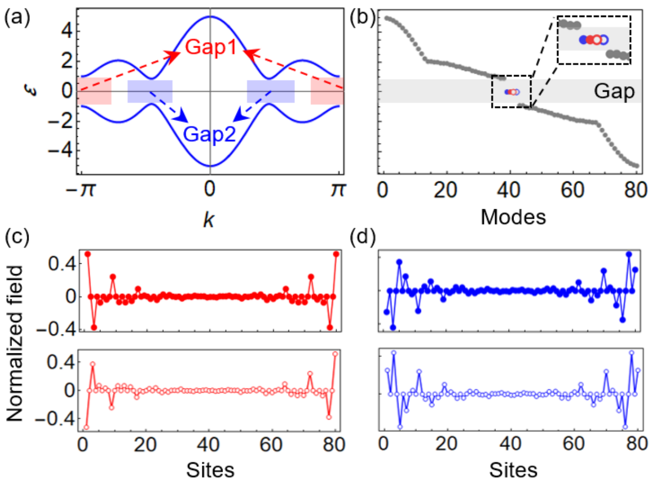

FIG. S1. (a) Band structure of the static model under PBC with c_{32}/c_{31}= 2, c_{31}=c_{10}. (b) Corresponding mode diagram under OBC with 80 sites. (c) and (d) Normalized field of topological zero modes within the bandgap corresponding to (b). (c) is the traditional zero modes while (d) is long-range coupling induced zero edge states.

Section II. Experimental proposals.

Here, we would like to propose a possible experimental realization of the Floquet waveguide lattice with long-range coupling. The 3rd NN couplings of our model [with $c_{32}(z) \equiv 0$] can be rearranged into a ladder waveguide lattice shown in Fig. S2. The waveguides are periodically bent in the x-z plane along their propagating direction z with period P modulating the driven frequency. The average coupling amplitudes c_{10} and c_{30} are controlled by distances d_1, d_2, d_3, and $\delta c_1(\delta c_3)$ is introduced by the bending amplitude A. Such waveguide lattice can be fabricated by the femtosecond-laser direct-writing technique [3-5].

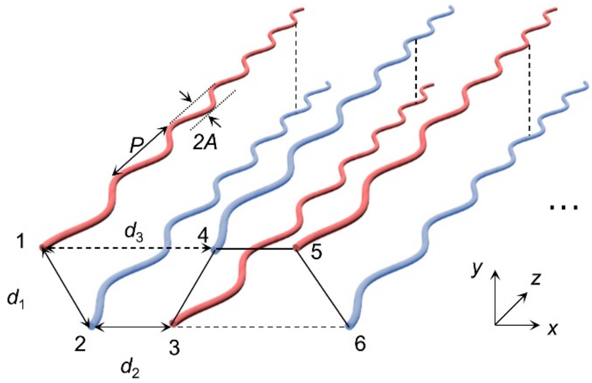

FIG. S2. Possible realization in the ladder waveguide lattice.

Section III. Floquet replica analysis.

We rewrite the Hamiltonian [Eq. (1) in the main text] as a sum of z-independent and z-periodic parts:

$$H(z) = H_0 + H_P(z), \tag{S1}$$

where

$$H_0 = \sum_{j=1}^{N} \left(c_{10} a_{B,j}^{\dagger} a_{A,j} + c_{10} a_{A,j+1}^{\dagger} a_{B,j} + c_{30} a_{B,j+1}^{\dagger} a_{A,j} + c_{30} a_{A,j+2}^{\dagger} a_{B,j} \right) + h.c., \tag{S2}$$

and

$$H_P(z) = \sum_{j=1}^{N} \begin{matrix} \left[-\delta c_1 \cos(wz+\varphi)\right] a_{B,j}^{\dagger} a_{A,j} + \left[\delta c_1 \cos(wz+\varphi)\right] a_{A,j+1}^{\dagger} a_{B,j} \\ + \left[\delta c_1 \cos(wz+\varphi)\right] a_{B,j+1}^{\dagger} a_{A,j} + \left[-\delta c_1 \cos(wz+\varphi)\right] a_{A,j+2}^{\dagger} a_{B,j} \end{matrix} + h.c.. \tag{S3}$$

Here, c_{10} and c_{30} in Eq. (S2) are the initial NN and 3rd NN coupling without bending, respectively, δc_1 and δc_3 denote the amplitudes of modulation and ω ($\omega \equiv 2\pi/P$, P is the period) is the modulation frequency. φ is the initial phase determined by the starting distance z = 0. Floquet theory [6] can be applied to analyze this z-periodic Hamiltonian $H(z+P)=H(z)$ with a period P. The solution of the equation $i\frac{\partial}{\partial z}|\psi(z)\rangle = H(z)|\psi(z)\rangle$ can be written as a superposition of Floquet states:

$$|\psi_\alpha(z)\rangle = \exp(-i\varepsilon_\alpha z)|u_\alpha(z)\rangle, \tag{S4}$$

where ε_α is the quasienergy and $|u_\alpha(z)>$ is the associated Floquet mode. The Floquet modes are P-periodic functions and belong to extended Hilbert space, which is a direct product of the usual Hilbert space and the space of z-periodic functions with period P. Substituting the Floquet ansatz [Eq. (S4)] into equation, we arrive at the eigenvalue equation:

$$(H(z) - i\frac{\partial}{\partial z})|u_\alpha(z)\rangle = \varepsilon_\alpha |u_\alpha(z)\rangle. \tag{S5}$$

Based on the periodic property, the Hamiltonian and Floquet modes can be spectrally decomposed as

$$H(z)=\sum_{n=-\infty}^{\infty} e^{-in\omega z} H_n, \tag{S6}$$

$$\left|u_\alpha(z)\right\rangle=\sum_{n=-\infty}^{\infty} e^{-in\omega z}\left|u_\alpha^n\right\rangle, \tag{S7}$$

where non-zero components H_0 and $H_{\pm1}$ are

$$\begin{aligned} H_0 &= \left[c_{10}+c_{10}\cos(k)+c_{30}\cos(k)\right]\sigma_x \\ &+\left[c_{10}\sin(k)-c_{30}\sin(k)\right]\sigma_y, \end{aligned} \tag{S8}$$

$$\begin{aligned} H_{\pm1} &= \frac{1}{2}\left[-\delta c_1 e^{\pm i\varphi}+\delta c_1 e^{\pm i\varphi}\cos(k)-\delta c_3 e^{\pm i\varphi}\cos(k)\right]\sigma_x \\ &+\left[\delta c_1 e^{\pm i\varphi}\sin(k)+\delta c_3 e^{\pm i\varphi}\sin(k)\right]\sigma_y, \end{aligned} \tag{S9}$$

under the periodic boundary condition. Here k is the Bloch wavevector and $\sigma_{x,y}$ correspond to Pauli matrices. The z-independent Floquet equation can be obtained by substituting these expansions into Eq. (S5):

$$\left(H_0-n\omega\right)\left|u_\alpha^n\right\rangle+\sum_{m\neq0} H_m\left|u_\alpha^{n-m}\right\rangle=\varepsilon_\alpha\left|u_\alpha^n\right\rangle, \forall n\in \mathrm{Z}. \tag{S10}$$

The Floquet band structure can be obtained by solving the above equation and truncating at a large n (as shown in Fig. 3 in the main text). The calculated band structure with long-range coupling clearly shows the closing and reopening of π gaps, indicating the topological phase transition and the emergence of the π modes.

Section IV. Topological invariant with long-range coupling.

In a Floquet system, the topological feature of π gap originates from the interaction of different Floquet replica bands, which is indicated by the topological invariant G_π in a chiral symmetric system [i.e. $\sigma_z H(z, k)\sigma_z = -H(-z, k)$] [7]. In our case, G_π can be calculated by the time evolution operator $U(z)$ and z-averaged effective Hamiltonian H_{eff}. By decomposing $U(z)$ and H_{eff} as $U(z) = \prod U(z, k)$ and $H_{\text{eff}} = \sum_k H_{\text{eff}}(k)$, G_π writes [7]

$$G_\pi=\frac{i}{2\pi}\int_{-\pi}^{\pi} tr[\left(V_\pi^+\right)^{-1}\partial_{\mathrm{k}} V_\pi^+]dk, \tag{S11}$$

where V_π^+ can be obtained from $V(z, k)$ at half period:

$$V(z,k)\equiv U(z,k)e^{iH_{eff}(k)z}, \tag{S12}$$

$$V\left(\frac{P}{2},k\right)=\begin{pmatrix} V_\pi^+ & 0 \\ 0 & V_\pi^- \end{pmatrix}. \tag{S13}$$

According to the bulk-edge correspondence in the Floquet system, G_π indicates the number of topological modes inside the corresponding π gap. In contrast to previous works which only consider the nearest-neighbor coupling, our work shows that long-range coupling can also effectively modulate the band structure. The interaction of replica bands and the topological features become more complex and fruitful with the presence of long-range coupling, which give rise to a larger value of G_π and indicate the emergence of new π modes.

Section V. Floquet topological phases with complex long-range coupling.

In the main text, we only consider the long-range coupling $c_{31}(z)$ and set $c_{32}(z)\equiv 0$ in the calculation to simplify the model. However, for more general cases with non-zero $c_{32}(z)$, FTP with $G_\pi>2$ and more π modes can be expected. The calculated topological invariant G_π with $c_{32}(z)\neq 0$ is shown in Fig. S3, and there are four topological regions indicated by different G_π.

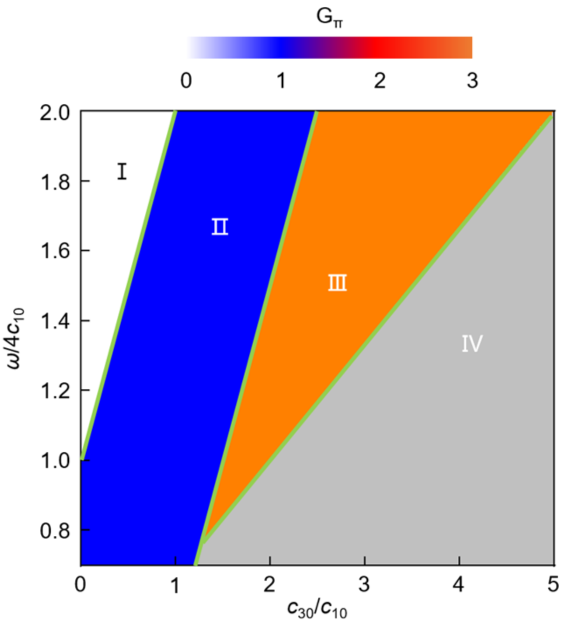

FIG. S3. Calculated G_π and phase diagram of π mode as a function of $\omega/4c_{10}$ and c_{30}/c_{10} with $c_{32}\neq 0$. Green lines indicate the boundary of different topological phases. $G_\pi = 3$ in the region III while it shows non-integer value in region IV.

G_π is equal to zero in region I and one in region II, which are similar to the case of $c_{32}(z)=0$ shown in the main text. However, G_π jumps to three in region III, indicating a quite novel topological phase induced by replica band interaction with $c_{32}(z)\neq 0$, which has no counterpart in the case of $c_{32}(z)=0$. Further increasing the long-range coupling strength leads to complex replica band interactions and results in non-integer value of G_π, similar to the case of low-frequency region mentioned in Refs. [8,9].

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