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Supplement DOI: https://doi.org/10.6084/m9.figshare.23628933

Parent Article DOI: https://doi.org/10.1364/OPTICA.497170

Natural exceptional points in the excitation spectrum of a light-matter system: Supplementary information

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Analysis of linear excitations

The effective non-Hermitian Hamiltonian described in the main text can be obtained using the approximation of linearized excitations, considering small fluctuations of the photonic δ_C and excitonic δ_X fields around their steady states ψ_C^{ss} and ψ_X^{ss}. The derivation goes is analogous to the derivation of Bogoliubov-de Gennes modes in an interacting Bose gas. In our analysis, the mean-field photonic and excitonic polariton components were

$$\begin{aligned} \psi_C(\mathbf{r},t) &= \psi_C^{ss} e^{-i\omega_p t}(1+\delta_C(\mathbf{r},t)), && \text{(SA.1)} \\ \psi_X(\mathbf{r},t) &= \psi_X^{ss} e^{-i\omega_p t}(1+\delta_X(\mathbf{r},t)). && \text{(SA.2)} \end{aligned}$$

Substituting Eqs. (SA.1) and (SA.2) into the system of Eqs. (2) and (3), the evolution of particle-like δ_X and δ_C and hole-like excitations, δ_X^* and δ_C^* were derived

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\delta_C &= (\hat{E}_C - \hbar\omega_p - i\hbar\gamma_C)\delta_C + \tfrac{\hbar\tilde{\Omega}_R}{2}\tfrac{\psi_X^{ss}}{\psi_C^{ss}}\delta_X, && \text{(SA.3)} \\ i\hbar\frac{\partial}{\partial t}\delta_C^* &= -(\hat{E}_C - \hbar\omega_p + i\hbar\gamma_C)\delta_C^* - \tfrac{\hbar\tilde{\Omega}_R}{2}\tfrac{\psi_X^{ss}}{\psi_C^{ss}}\delta_X^*, && \text{(SA.4)} \end{aligned}$$

$$\begin{aligned} i\hbar\frac{\partial}{\partial t}\delta_X &= (\hat{E}_X - \hbar\omega_p - i\hbar\gamma_X)\delta_X + \tfrac{\hbar\tilde{\Omega}_R}{2}\tfrac{\psi_C^{ss}}{\psi_X^{ss}}\delta_C + g_X(\psi_X^{ss})^2(2\delta_X + \delta_X^*), && \text{(SA.5)} \\ i\hbar\frac{\partial}{\partial t}\delta_X^* &= -(\hat{E}_X - \hbar\omega_p + i\hbar\gamma_X)\delta_X^* - \tfrac{\hbar\tilde{\Omega}_R}{2}\tfrac{\psi_C^{ss}}{\psi_X^{ss}}\delta_C^* - g_X(\psi_X^{ss})^2(2\delta_X^* + \delta_X). && \text{(SA.6)} \end{aligned}$$

Keeping only the linear terms, Eqs. (SA.3)–(SA.6) can be written in a matrix form as

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\delta_C(t,k)\\ \delta_X(t,k)\\ \delta_C^*(t,k)\\ \delta_X^*(t,k)\end{pmatrix} = \begin{pmatrix} E_C(k)-\hbar\omega_p-i\hbar\gamma_C & \frac{\hbar\tilde{\Omega}_R}{2}\frac{\psi_X^{ss}}{\psi_C^{ss}} & 0 & 0 \\ \frac{\hbar\tilde{\Omega}_R}{2}\frac{\psi_C^{ss}}{\psi_X^{ss}} & E_X(k)-\hbar\omega_p-i\hbar\gamma_X+2g_X(\psi_X^{ss})^2 & 0 & g_X(\psi_X^{ss})^2 \\ 0 & 0 & -E_C(k)+\hbar\omega_p-i\hbar\gamma_C & -\frac{\hbar\tilde{\Omega}_R}{2}\frac{\psi_X^{ss*}}{\psi_C^{ss*}} \\ 0 & -g_X(\psi_X^{ss})^2 & -\frac{\hbar\tilde{\Omega}_R}{2}\frac{\psi_C^{ss*}}{\psi_X^{ss*}} & -E_X(k)+\hbar\omega_p-i\hbar\gamma_X-2g_X(\psi_X^{ss})^2 \end{pmatrix}\begin{pmatrix}\delta_C(t,k)\\ \delta_X(t,k)\\ \delta_C^*(t,k)\\ \delta_X^*(t,k)\end{pmatrix}. \tag{SA.7}$$

Eigenenergies of the matrix are equivalent to the spectrum of elementary excitations of the system. Assuming that the interaction strength between excitons is negligibly small ($g_X \approx 0$), considering the part of the spectrum near the exceptional point where $\frac{\psi_C^{ss}}{\psi_X^{ss}} \approx \frac{\psi_X^{ss}}{\psi_C^{ss}} \approx 1$, $\frac{\psi_C^{ss*}}{\psi_X^{ss*}} \approx \frac{\psi_X^{ss*}}{\psi_C^{ss*}} \approx 1$, Eq. (SA.7) takes the form

$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\delta_C(t,k)\\ \delta_X(t,k)\\ \delta_C^*(t,k)\\ \delta_X^*(t,k)\end{pmatrix} = \begin{pmatrix} E_C(k)-\hbar\omega_p-i\hbar\gamma_C & \frac{\hbar\tilde{\Omega}_R}{2} & 0 & 0 \\ \frac{\hbar\tilde{\Omega}_R}{2} & E_X(k)-\hbar\omega_p-i\hbar\gamma_X & 0 & 0 \\ 0 & 0 & -E_C(k)+\hbar\omega_p-i\hbar\gamma_C & -\frac{\hbar\tilde{\Omega}_R}{2} \\ 0 & 0 & -\frac{\hbar\tilde{\Omega}_R}{2} & -E_X(k)+\hbar\omega_p-i\hbar\gamma_X \end{pmatrix}\begin{pmatrix}\delta_C(t,k)\\ \delta_X(t,k)\\ \delta_C^*(t,k)\\ \delta_X^*(t,k)\end{pmatrix}, \tag{SA.8}$$

The block structure of the above matrix reflects the “normal” and “ghost” branches of the Bogoliubov excitation spectrum of elementary excitations in the system. Focusing on the normal branch, for δ_C and δ_X we recover the spectrum of the non-Hermitian Hamiltonian described by Eq. (5).

Collective excitation spectrum

Bogoliubov quasiparticles are collective excitations emerging in superconductors and superfluids. Their signatures in bosonic condensates include the characteristic linear dependence between the energy of elementary excitation and the wave vector[1]. Additionally, Bogoliubov excitation spectrum is symmetric with respect to vacuum energy, which results in the appearance of a virtual energy branch, the so-called "ghost branch".

According to the Landau criterion, a system with such a spectrum behaves like a superfluid. Therefore, observation of Bogoliubov excitation was essential for understanding the nature of bosons scattering, interactions, and their condensates stability.

In the case of nonequilibrium, incoherently pumped exciton-polariton condensates, Bogoliubov excitation has a more complex character than in the equilibrium case. The nontrivial shape of the spectra is induced by the interaction with an excitonic reservoir. Such an interaction can be observed even in resonantly excited systems[2–5]. During the past several years, these interesting observations have stimulated numerous questions about the nature of collective excitations in polariton condensates.

According to the theoretical framework presented in a recent work[5], these effects can result from two-body interactions. This work experimentally confirmed that the excitation observed in such a system exhibits a hybrid nature connecting properties of the Bogoliubov and reservoir density excitations. The appearance of an additional excitonic reservoir can drastically affect the superfluid properties and change the character of the quantum fluid spectrum.

Here, we want to point out some similarities between our experimental observation and the results of the previous work[5]. Similarly as in the previous work[5], we assume that thermally excited acoustic phonons generate an incoherent reservoir which interacts with polaritons. From the experimental point of view, the main difference between our results and the previous work is in the chosen material of the quantum well. Compared to the GaAs structure explored in the previous work, our CdTe-based microcavity is characterized by the lack of observable energy blueshift of polariton modes.

In result, the excitation spectrum in our case is substantially modified as the laser power is increased, with a visible reduction of coupling strength and polariton up-conversion. However, due to the weakness of interactions, the effects characteristic for the Bogoliubov quasiparticles, such as the linearized energy spectrum at low momenta and the ghost branch, cannot be detected.

It should be noted that the exact physical mechanism for the observed up-conversion, and the role of biexciton or dark exciton reservoir are under debate and will stimulate further discussion on this subject.

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